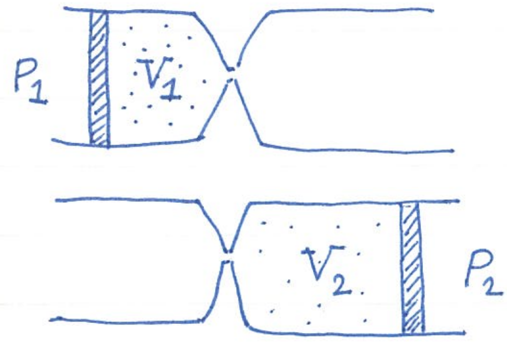
HH0039 Joule-Thomson Effect

There are various ways to cool down the temperature, working at different regimes. In the notes, I will give brief introduction to (1) Joule-Thomson cooling (2) Helium dilution refrigerator (3) Isentropic demagnetization (4) Laser Doppler cooling.

⓪ Joule-Thomson cooling:



Assuming no work is done through the pinch hole, the change of internal energy equals the work done at constant pressure (no heat flow, $Q=0$)

$$U_2 - U_1 = P_1 V_1 - P_2 V_2$$

Recall the definition of enthalpy $H = U + pV$. It is clear that the enthalpy remains constant during the expansion. $H_1 = H_2$

For ideal gas, $pV = N\tau \longrightarrow H = \frac{5}{2} N\tau$.

Thus, $\tau_1 = \tau_2$, temperature remains the same before/after expansion.

What about realistic gas? Consider van der Waals gas,

$$\underbrace{U = \frac{3}{2} N\tau - \frac{N^2 a}{V}}_{\text{attraction}} \quad \text{and} \quad p = \frac{N\tau}{V - Nb} - \frac{N^2 a}{V^2} \approx \underbrace{\frac{N\tau}{V} + \frac{N^2}{V^2} b\tau - \frac{N^2 a}{V^2}}_{\text{expansion to } O(a,b)}$$

The enthalpy to the lowest order in a, b is

$$\boxed{H = U + pV = \frac{5}{2} N\tau + \frac{N^2}{V}(b\tau - 2a)} \longrightarrow \tau_{inv} = \frac{2a}{b} = \frac{27}{4}\tau_c$$

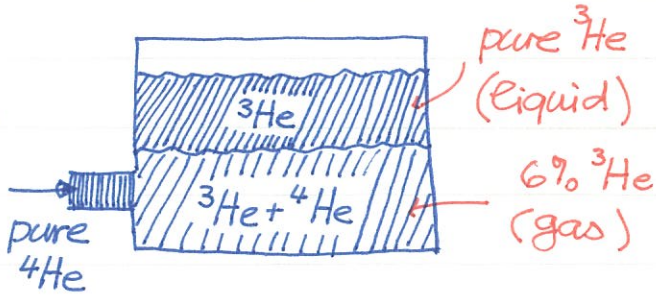
Rewriting the enthalpy $H = H_0 + \frac{N^2}{V} b(\tau - \tau_{inv})$, it's easy to derive

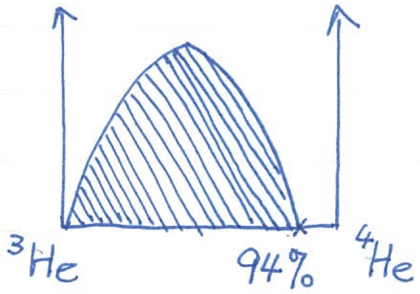
$$\underbrace{H(\tau) = g(V)\tau + C_2}_{\text{linear dependence in } \tau} \xrightarrow{\text{plot!}} \begin{cases} \tau < \tau_{inv}, & \tau_2 < \tau_1 \quad \text{cooling!} \\ \tau > \tau_{inv}, & \tau_2 > \tau_1 \quad \text{heating!} \end{cases}$$

For $\tau < \tau_{inv}$, temperature cools down after expansion due to inter-molecular attraction. Q: Why does heating occur when $\tau > \tau_{inv}$? The attraction is still there.... ?

⊘ Helium dilution refrigerator:

Consider the ^3He, ^4He mixture as shown on the left. Recall the phase diagram for Helium —





At $T \approx 0$,
the setup is stable for 94% ^4He and 6% ^3He mixture. When pure ^4He pumped into the binary mixture, ^3He concentration is diluted. To maintain the equilibrium, some of the pure ^3He will "evaporate" into the binary mixture. Due to the latent heat, the temperature is cooled.

One may think 5% ^3He + 95% ^4He binary mixture is also stable in phase diagram. Why is the 6% ^3He concentration necessary? Well, according to the equilibrium conditions in binary mixture,

$$\boxed{\mu(^3He) = \mu(6\%\ ^3He + 94\%\ ^4He)}$$

Therefore, as long as ^3He is present, 6% ^3He mixture is required ☺

⊘ Isentropic demagnetization:

$E = -\vec{m} \cdot \vec{B}$

Consider the Ising limit $\vec{m} = \pm m\,\hat{z}$, the average magnetization at temperature τ is

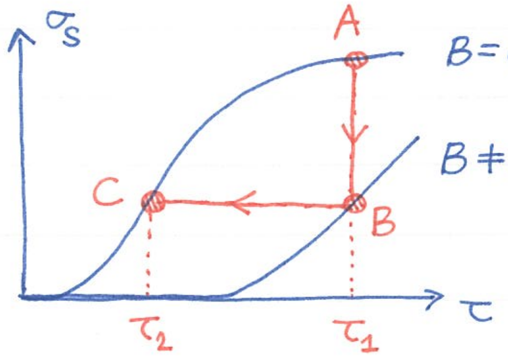
$$\boxed{M = Nm \tanh\left(\frac{mB}{\tau}\right)}$$

variable $\frac{mB}{\tau}$!

The spin polarization $P = \tanh(mB/\tau)$ and the probability distribution is

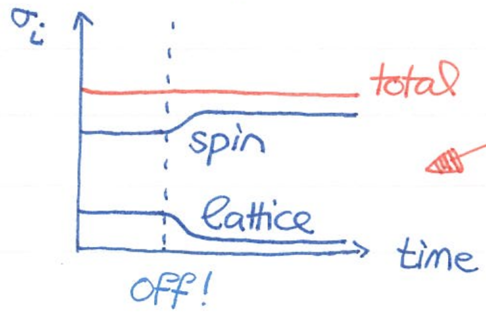
$$P_\uparrow = \frac{1}{2}(1+P)\ ,\quad P_\downarrow = \frac{1}{2}(1-P)$$

The spin entropy is $\sigma_s = -P_\uparrow \log P_\uparrow - P_\downarrow \log P_\downarrow$



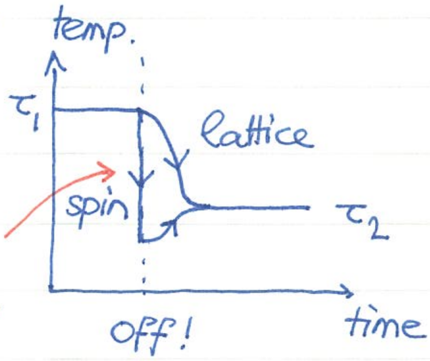
By applying a strong field to align the spins (A → B), then turn off the magnetic field adiabatically, the temperature cools from τ_1 to τ_2. (B → C). The trick is to remove spin entropy by external magnetic field.

However, the trick needs some mechanism to "thermalize" the spin configurations: lattice-spin interactions. During the isentropic demagnetization (B → C) process, the whole system is not in thermal equilibrium during transient period.



$\sigma_{total} = \sigma_{spin} + \sigma_{lattice}$

spin, lattice not in equilibrium.



Suppose the internal magnetic field is B_Δ (even at $\vec{B}=0$), the temperatures τ_2, τ_1 are related by the constant entropy

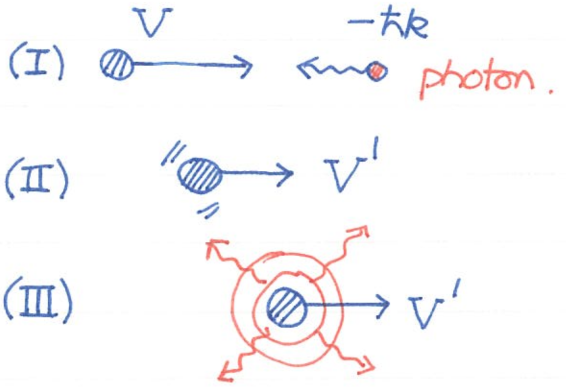
$$\sigma = \sigma_{spin} + \sigma_{lattice} = \text{const} \longrightarrow \boxed{\sigma_{spin} = \sigma_{spin}\left(\frac{mB}{\tau}\right) \approx \text{const}}$$

Therefore, one arrives at the simple relation:

$$\frac{B_\Delta}{\tau_1} = \frac{B}{\tau_2} \longrightarrow \boxed{\tau_2 = \left(\frac{B_\Delta}{B}\right)\tau_1}$$ where $B_\Delta < B$ is assumed.

So, we can cool the spin system by turning on/off $\vec{B}$ field ☺

⑦ Laser Doppler cooling: Nobel Prize 1997, a useful reference can be found in Reviews of Modern Physics 70, 721 (1998).
Choose a laser on resonance with some atom.

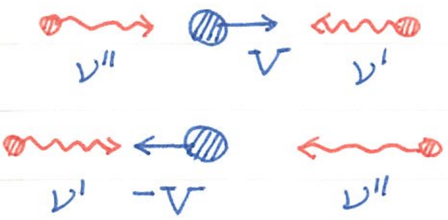


The atom absorbs the photon

$$V' = \frac{p'}{M} = \frac{p-\hbar k}{M} = V - \frac{\hbar k}{M} < V$$

It slows down by radiation pressure!
The photon-emission process is isotropic and does not change the average velocity V'!

Doppler cooling...



Due to Doppler effect, the photon frequencies are shifted,

$$\boxed{\begin{aligned} \nu' &= \nu_0 \sqrt{\frac{c+v}{c-v}} > \nu_0 \\ \nu'' &= \nu_0 \sqrt{\frac{c-v}{c+v}} < \nu_0 \end{aligned}}$$

→ on resonance.
→ off resonance.

The atom always slow down no matter which direction it moves — just like friction ☺

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