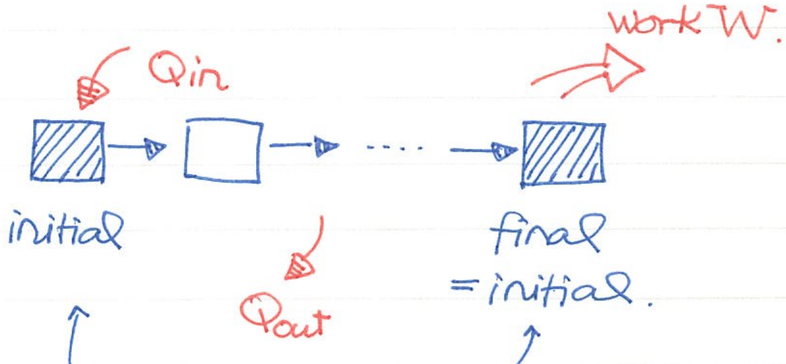
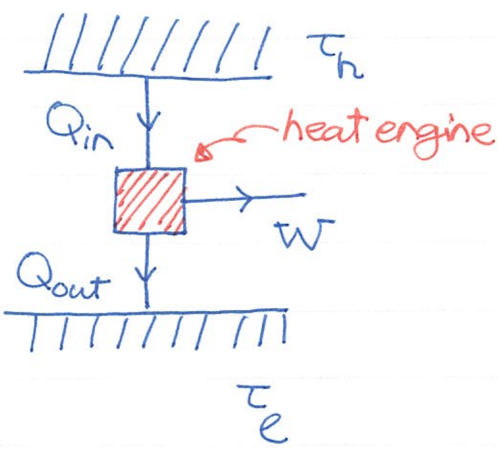
HH0017 Heat Engine

What's a heat engine?

It's a cyclic device converting heat Q_{in} into work W. It's convenient to use the heat flow diagram:



It's important to have the cyclic property.

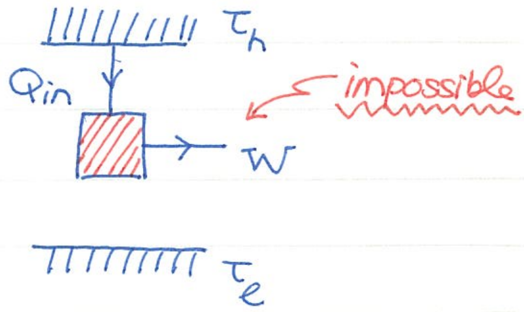


> Kelvin-Plank form of the 2nd law: It is impossible to convert heat into work without any loss for cyclic processes.

The statement is quite simple in terms of diagram.

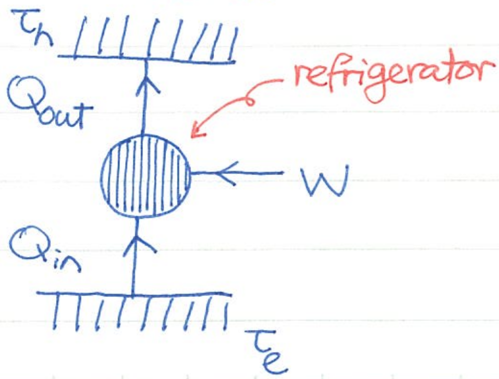
Introduce the efficiency of heat engine,

$$\boxed{\eta \equiv \frac{W}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}}$$



Because $Q_{out} > 0$, the efficiency $\eta < 1$.

If we reverse the cyclic processes of a heat engine, we get a refrigerator. The point is to pump heat Q_{in} from low temperature to high temperature by applying external work W

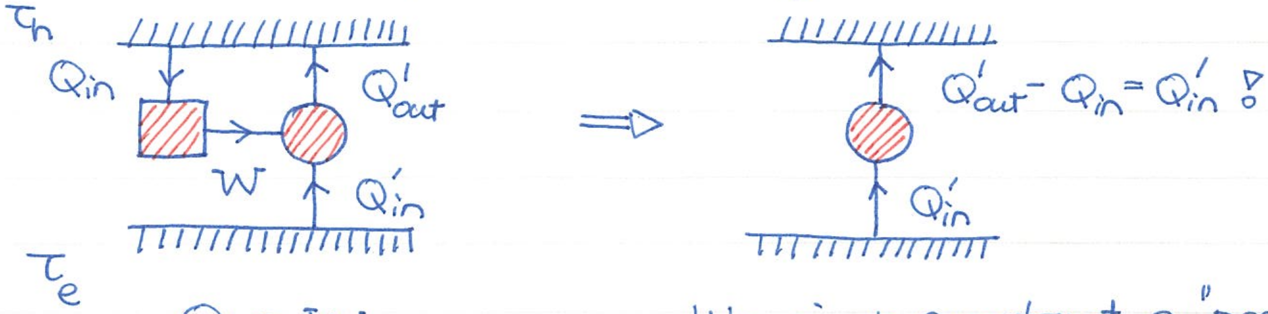


$$\boxed{\gamma \equiv \frac{Q_{in}}{W}}$$

note that γ is not necessarily smaller than unity .....

C.O.P. : coefficient of performance

If we have a "perfect" heat engine, ...

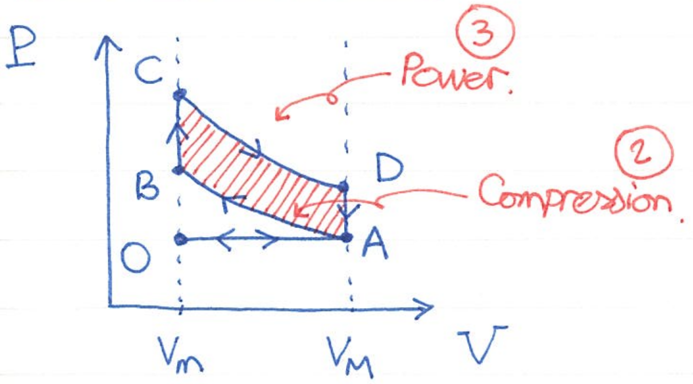


$$\begin{cases} Q_{in} = W \\ Q'_{out} = Q'_{in} + W \end{cases}$$

We just construct a "perfect" refrigerator ⊙⊙ Heat flows from τ_e to τ_h without applying any external work.

contradict the 2^{nd} law.

$\boxed{\text{② Otto cycle :}}$ 4-step engine commonly used. During $B \to C$, Q_{in} flows in while Q_{out} flows out during $D \to A$.



$$\boxed{\begin{aligned} Q_{in} &= C_V(\tau_C - \tau_B) \\ Q_{out} &= C_V(\tau_D - \tau_A) \end{aligned}}$$

O → A : Intake. ①

O ← A : Exhaust. ④

Note that $A \to B$ and $C \to D$ are adiabatic processes w/o heat flows.

efficiency $\eta = 1 - \dfrac{Q_{out}}{Q_{in}} = 1 - \dfrac{\tau_D - \tau_A}{\tau_C - \tau_B}$ too complicated ...

Making use of the relation $\tau V^{\gamma-1} = \text{const}$ for adiabatic process,

$$\frac{\tau_D}{\tau_C} = \left(\frac{V_C}{V_D}\right)^{\gamma-1} = \left(\frac{V_m}{V_M}\right)^{\gamma-1}$$

similarly

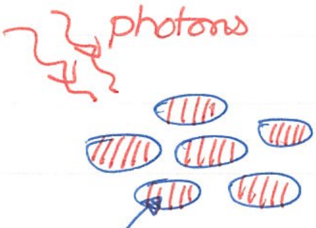
$$\frac{\tau_A}{\tau_B} = \left(\frac{V_B}{V_A}\right)^{\gamma-1} = \left(\frac{V_m}{V_M}\right)^{\gamma-1}$$

$$\Longrightarrow \boxed{\eta = 1 - \left(\frac{V_m}{V_M}\right)^{\gamma-1}}$$

For a realistic engine, $\gamma \approx 1.4$ and $V_m/V_M \approx 1/8$, giving the ideal efficiency $\eta \approx 56\%$ $\longrightarrow$ 15% - 20%

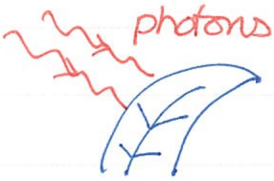
due to friction, heat loss, ...

Think about other ways to generate energies in useful forms?



solar cell's

photovoltaics



photosynthesis

Efficiencies for both processes are compared in a recent article in Science:

$\boxed{\text{Science } \underline{332}, 805 (2011).}$

You may also find "quantum coherence" in photosynthesis:

Nature $\underline{446}$, 782 (2007)
Nature $\underline{463}$, 644 (2010)

$\boxed{\text{What's the difference between heat and work?}}$

According to the 1st law. $\Delta U = Q + W$

no entropy changes

Both Q and W are energy transfer. The difference lies in entropy association

associated with entropy change.

Why? Starting from $U = U(\sigma, V)$

no entropy change.

$$dU = \left(\frac{\partial U}{\partial \sigma}\right)_V d\sigma + \left(\frac{\partial U}{\partial V}\right)_\sigma dV = \underbrace{\tau d\sigma}_{\text{heat}} - pdV$$

Sometimes, the above relation is written as $dU = đQ + đW$

Another interesting derivation is the following

$$U \equiv \langle \varepsilon_s \rangle = \sum_s P_s \varepsilon_s \longrightarrow dU = \underbrace{\sum_s \varepsilon_s dP_s}_{\text{probability distribution}} + \underbrace{\sum_s P_s d\varepsilon_s}_{\text{energy levels.}}$$

The entropy can be expressed in terms of the probability distribution,

$$\sigma = \sum_s -P_s \log P_s \longrightarrow d\sigma = \sum_s -\log P_s \, dP_s - \cancel{P_s} \frac{1}{\cancel{P_s}} dP_s$$

Assuming it's Boltzmann distribution $P_s = \frac{1}{Z} e^{-\varepsilon_s/\tau}$

$$d\sigma = \sum_s \frac{\varepsilon_s}{\tau} dP_s + (\log Z - 1) \underbrace{\sum_s dP_s}_{\text{probability conservation.}}$$

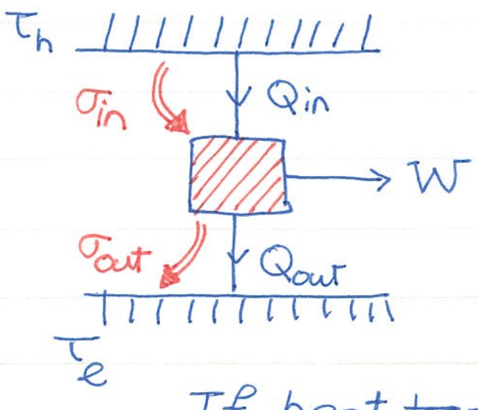
$$\longrightarrow \boxed{\tau d\sigma = \sum_s \varepsilon_s dP_s}$$ change of the probability distribution

It's straightforward to see $\sum_s P_s d\varepsilon_s = \sum_s P_s \frac{d\varepsilon_s}{dV} \cdot dV$

$$\longrightarrow \boxed{-p dV = \sum_s P_s d\varepsilon_s}$$ change of the energy levels !

⑦ reversible and irreversable processes

Let's try to add in entropy change. The problem is ... the entropy is not conserved.



(i) If only reversible processes are involved, $\boxed{\sigma_{in} = \sigma_{out}}$

(ii) If there are irreversible processes, $\boxed{\sigma_{in} < \sigma_{out}}$

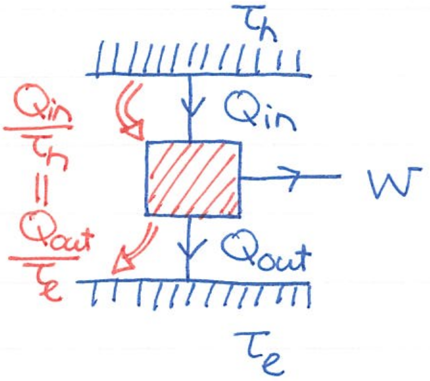
If heat transfers occur at constant temperatures, the entropy changes σ_{in} and σ_{out} can be computed easily.

$\sigma_{in} = \dfrac{Q_{in}}{\tau_h}$ and $\sigma_{out} = \dfrac{Q_{out}}{\tau_e}$ If there's no irreversible process, we obtain the relation $\sigma_{in} = \sigma_{out}$

$$\Rightarrow \boxed{\frac{Q_{in}}{Q_{out}} = \frac{\tau_h}{\tau_e}}$$

must be so. otherwise the entropy will accumulate during the cyclic processes.

The efficiency is $\eta = 1 - \dfrac{Q_{out}}{Q_{in}} = 1 - \dfrac{\tau_e}{\tau_h}$



This ideal cyclic process is the famous Carnot cyclic with efficiency only depending on the temperature ratio.

$$\boxed{\eta = 1 - \frac{\tau_e}{\tau_h}}$$

$\eta \to 1$ as $\tau_e \to 0$, the absolute zero temperature.

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