微積分（A）附件

【章節 15.6】(Limit and Continuity)

※補充題用

1.Find the limit, if it exists or show that the limit does not exists.

ⓐ$\lim_{(x,y)\to(5,-2)}(x^5+4x^3y-5xy^2)$

ⓑ$\lim_{(x,y)\to(6,3)} xy\cos(x-2y)$

ⓒ$\lim_{(x,y)\to(0,0)}\frac{y^4}{x^3+3y^4}$

ⓓ$\lim_{(x,y)\to(0,0)}\frac{x^2+\sin^2 y}{2x^2+y^2}$

ⓔ$\lim_{(x,y)\to(0,0)}\frac{xy^4}{x^2+y^8}$

ⓕ$\lim_{(x,y)\to(0,0)}\frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1}$

ⓖ$\lim_{(x,y)\to(0,0)}\frac{x^4-y^4}{x^2+y^2}$

2.Determine the set of the points at which the function is cont.

ⓐ$F(x,y)=\frac{\sin(xy)}{e^x-y^2}$　　ⓑ$f(x,y)=\begin{cases}\frac{x^2y^3}{2x^2+y^2}, & \text{if}(x,y)\neq(0,0)\\ 1, & \text{if}(x,y)=(0,0)\end{cases}$

3.Show that $f: D\subset\mathbb{R}^n\to\mathbb{R}$ given by $f(x)=\|x\|$ is cont. on $\mathbb{R}^n$.

4.Let $c\in\mathbb{R}^n$, show that the function $f(x)=c\cdot x\ \ \forall x\in\mathbb{R}^n$ is cont. on $\mathbb{R}^n$.

5.Show that $\lim_{(x,y)\to(0,0)}\sqrt{|xy|}=0$ by definition.

【章節 15.4】(Partial derivative)&【章節 15.6】(Equality of Mixed Partials)

1.Find the first partial derivatives of the function.

ⓐ$f(x,y)=\frac{x-y}{x+y}$　　ⓑ$f(x,t)=\tan^{-1}(x\sqrt{t})$　　ⓒ$f(x,y)=\int_y^x\cos(t^2)dt$

ⓓ$f(x,y,z,t)=xyz^2\tan(yt)$　ⓔ$u=\sqrt{x_1^2+x_2^2+\cdots+x_n^2}$　ⓕ$z(x,y)=\frac{f(x)}{g(y)}$　ⓖ $z(x,y)=f\left(\frac{x}{y}\right)$

2.Find all the second partial derivatives of $u(s,t)=e^{-s}\sin t$

3.Let $u(r,\theta)=e^{r\theta}\sin\theta$.　Find $\frac{\partial^3 u}{\partial r^2\partial\theta}$.

4.Verify that the function $u(x,y,z)=\frac{1}{\sqrt{x^2+y^2+z^2}}$ satisfies $u_{xx}+u_{yy}+u_{zz}=0$

5.If $u(x_1,x_2,\dots x_n)=e^{a_1x_1+\cdots+a_nx_n}$, where $a_1^2+\cdots+a_n^2=1$. Show that $\frac{\partial^2 u}{\partial x_1^2}+\cdots+\frac{\partial^2 u}{\partial x_n^2}=u$.

6.Let $f(x,y)=\begin{cases}(x^2+y^2)\sin\left(\frac{1}{x^2+y^2}\right), & \text{if } (x,y)\neq(0,0)\\ 0, & \text{if}(x,y)=(0,0)\end{cases}$.

①Show that fx and fy exist on $\mathbb{R}^2$, but not cont. at (0,0).

②Compute fxy(0,0) and fyx(0,0).