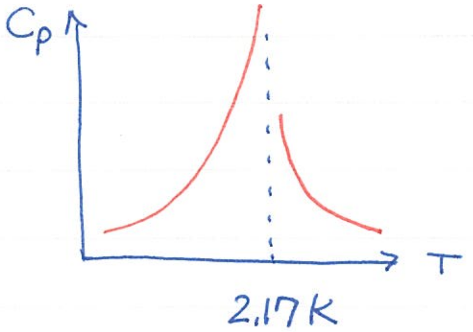
HH0016 Liquid Helium: Phase Diagram & Superfluidity.

The Einstein condensation temperature is defined as

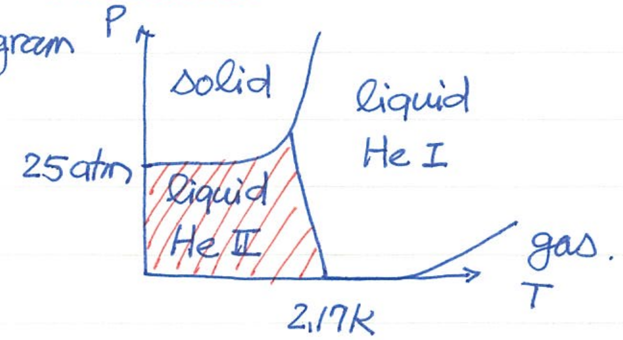
$$\boxed{\tau_E = \frac{2\pi\hbar^2}{m}\left(\frac{n}{2.612}\right)^{\frac{2}{3}}} \longrightarrow T_E = \frac{115}{V_M^{\frac{2}{3}} M}$$

T_E in K, V_M molar volume, M molecular weight

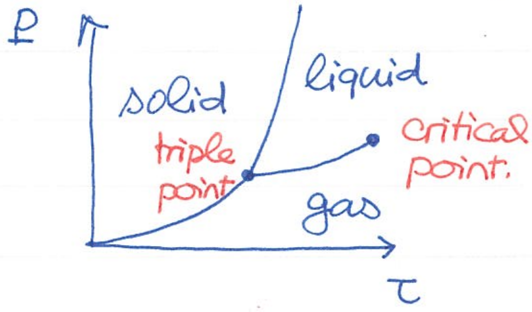
For liquid ^4He,
$V_M = 27.6\ cm^3/mole$ $M = 4$, thus $T_E = 3.1K$



phase diagram for ^4He



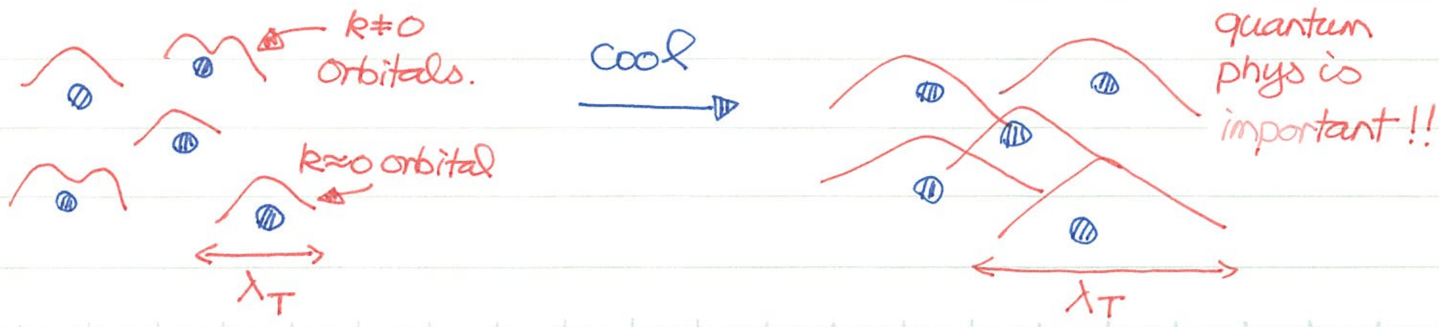
The phase diagram for ^4He is quite different from the ordinary one. In the low temp regime, He II phase is a superfluid!



For ordinary materials, phase diagram consists of three phases: solid, liquid, gas. Liquid and gas share the same symmetry and thus can transform into each other without going through any phase transition ☺

For ^4He atoms, one can introduce the thermal wavelength $\lambda_T = \frac{1}{n_Q^{1/3}}$

$$\lambda_T = \sqrt{\frac{2\pi\hbar^2}{m\tau}} \sim \frac{1}{\sqrt{T}}$$ When cooled down, λ_T increases!

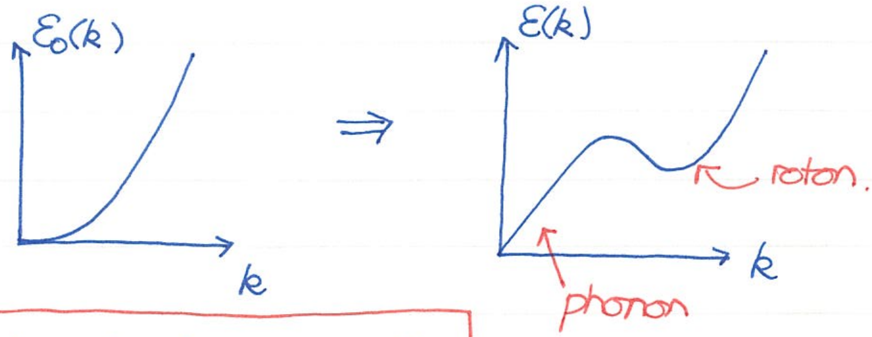


Almost all particles occupy the lowest orbital and the wave fn for the system is

$$\boxed{\Psi(x_1,\cdots x_N) \approx \phi(x_1)\phi(x_2)\cdots\phi(x_N)}$$

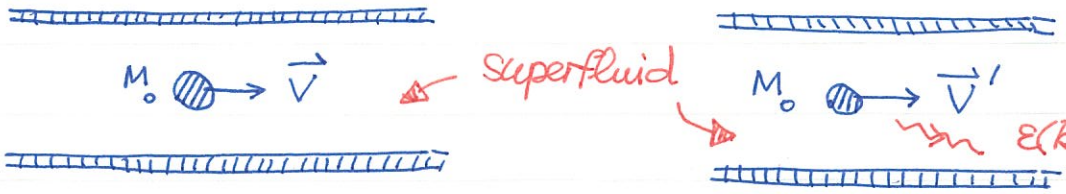
wave fn of the lowest orbital.

For simplicity, let's sit at $\tau=0$ so that all particles are at $k=0$ orbital. The energy is E_0 and the momentum is zero. It takes some energy $E_0+\varepsilon(\vec{k})$ to move the ground state to the excited state, now carrying momentum $\vec{k}$. These low-lying states are called elementary excitations. Sometimes, these excitations behave like particles with specific energy-momentum relation $\varepsilon(\vec{k})$ and are called quasiparticles.



He-He interactions change the dispersion from ε_0 to $\varepsilon(k)$!

$\boxed{\text{Landau's argument:}}$



$V<V_c$, no excitations $\rightarrow$ no damping ☺

$V>V_c$, velocity changes and excitations appear $\rightarrow$ damping.

$$\boxed{\begin{aligned}\tfrac{1}{2}M_0V^2 &= \tfrac{1}{2}M_0V'^2+\varepsilon(\vec{k})\\ M_0\vec{V} &= M_0\vec{V}'+\hbar\vec{k}\end{aligned}}$$

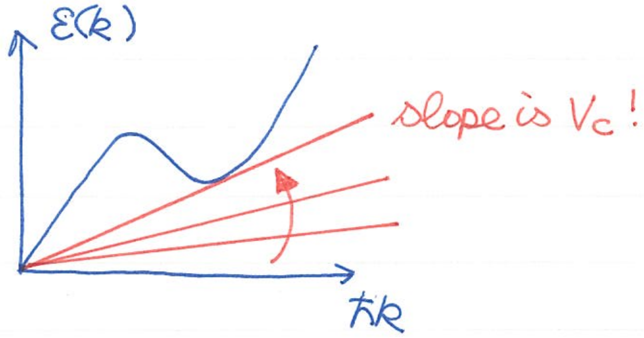
Both conservation laws must be satisfied to allowed an elementary excitation of $\varepsilon(\vec{k})$, $\hbar\vec{k}$.

$$(M_0\vec{V}-\hbar\vec{k})^2=(M_0\vec{V}')^2 \longrightarrow \tfrac{1}{2}M_0V^2+\frac{\hbar^2k^2}{2M_0}-\hbar\vec{V}\cdot\vec{k}=\tfrac{1}{2}M_0V'^2$$

combined with energy conservation $\boxed{\hbar\vec{V}\cdot\vec{k}=\varepsilon(\vec{k})+\frac{\hbar^2k^2}{2M_0}}$

To find the minimum V, it is clear that $\vec{V} \parallel \vec{k}$ $\longrightarrow$ $\vec{V}\cdot\vec{k} = V\cdot k$.
For realistic situation, M_0 is huge in comparison. $\longrightarrow$ drop $\hbar^2 k^2/2M_0$
Thus, the criterion is simple:

$$\hbar V_c k \cong \varepsilon(\vec{k}) \longrightarrow \boxed{V_c = \min\left\{ \frac{\varepsilon(\vec{k})}{\hbar k} \right\}}$$



From the graph, we know V_c is smaller than the speed of sound v_s.

It is also important to notice that $V_c = 0$ for non-interacting Bose gas with dispersion $\varepsilon_0(\vec{k}) = \hbar^2 k^2/2m$. Therefore, the condensate is not a superfluid. Luckily, we don't really have non-interacting bosons in nature ☺.

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