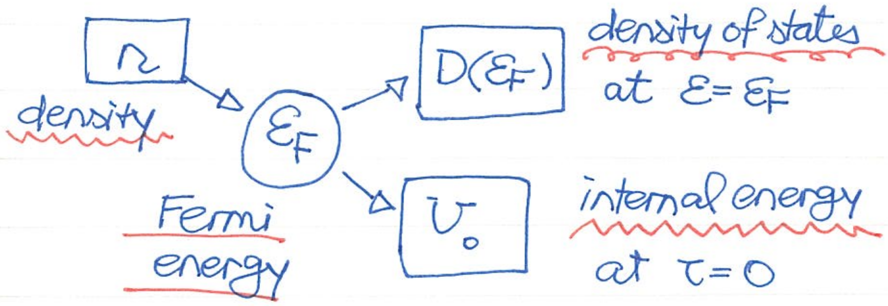
HH0015 Where Can You Find Fermi Gas?

Brief review on the non-interacting Fermi gas —

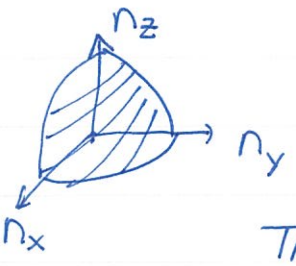


First of all, the energy is quantized.

$$\boxed{\varepsilon_n = \frac{\hbar^2}{2m}\left(\frac{n\pi}{L}\right)^2}$$

$n = (n_x, n_y, n_z)$

The total number of fermions is $N = (2s+1)\cdot\frac{1}{8}\cdot\frac{4\pi}{3}\,n_{max}^3$



Thus, $N = \frac{(2s+1)\pi}{6}\,n_{max}^3 = \frac{\pi}{3}\,n_{max}^3$ for $s=\frac{1}{2}$.

The Fermi energy is also related to n_{max} ☺

$$\varepsilon_F = \frac{\hbar^2}{2m}\left(\frac{n_{max}\pi}{L}\right)^2 = \frac{\hbar^2}{2m}\,\frac{\pi^2}{L^2}\cdot\left(\frac{3N}{\pi}\right)^{\frac{2}{3}} \longrightarrow \boxed{\varepsilon_F = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{\frac{2}{3}}}$$

At zero temperature, the internal energy can be expressed as

$$U_0 = \int_0^\infty d\varepsilon\cdot\varepsilon\, D(\varepsilon)\, f(\varepsilon) = \int_0^{\varepsilon_F} d\varepsilon\ \varepsilon\cdot D(\varepsilon_F)\frac{\sqrt{\varepsilon}}{\sqrt{\varepsilon_F}} = \frac{D(\varepsilon_F)}{\sqrt{\varepsilon_F}}\int_0^{\varepsilon_F} d\varepsilon\ \varepsilon^{\frac{3}{2}}$$

From previous lecture,

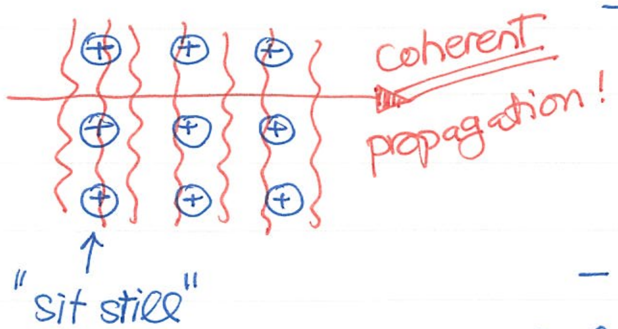
$$\boxed{D(\varepsilon_F) = \frac{3N}{2\varepsilon_F}} \longrightarrow \boxed{U_0 = \frac{3N}{2\varepsilon_F}\cdot\frac{1}{\sqrt{\varepsilon_F}}\cdot\frac{2}{5}\,\varepsilon_F^{\frac{5}{2}} = \frac{3}{5}N\varepsilon_F}$$

At $\tau = 0$, the average kinetic energy of each $s=\frac{1}{2}$ fermion is HUGE $\sim \frac{3}{5}\varepsilon_F$!!

⓪ Electrons in metals.

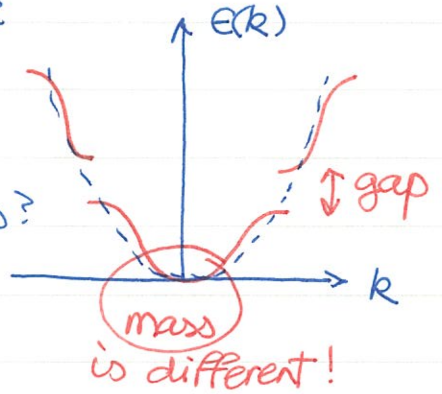
- (electron-ion) + (ion-ion) interactions.
- (electron-electron) interactions.

It's hard to believe that electrons can be treated as non-interacting Fermi gas ⊙⊙ But, they are indeed almost independent in metals .....

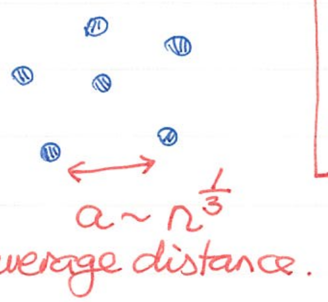


- Quantum physics tell us that an electron can propagate w/o difficulty as long as ions sit still without vibrations.
- But, there's visible consequence.

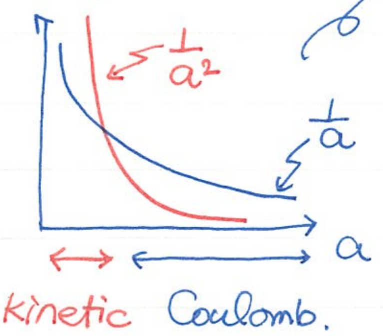
The mass of electrons is not the same as that of free electrons.



So, what about electron-electron interactions?



$$\boxed{\begin{aligned} &\text{Coulomb}: \frac{e^2}{a} \sim \frac{1}{a} \\ &\text{Kinetic}: \epsilon_F \sim \frac{1}{a^2} \end{aligned}}$$



Thus, when density is low (large a), Coulomb interaction dominate, electrons can form a crystal

Wigner crystal.

On the other hand, when density is high (small a), kinetic energy dominates and interactions are not as important.

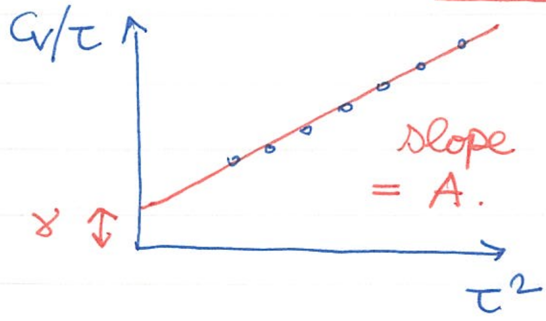
Estimate ε_F in metal: $\varepsilon_F = \frac{\hbar^2}{2m}(3\pi^2 n)^{\frac{2}{3}}$

For Cu: $n \sim 10^{23}\,cm^{-3} \longrightarrow \varepsilon_F \sim 7\,eV \sim 8.2\times10^4\,K$

meanwhile, $v_F \sim \frac{1}{100}c$ quite fast

Heat capacity $\boxed{C_V = \gamma\tau + A\tau^3}$ → phonon contribution.

→ electron contribution.

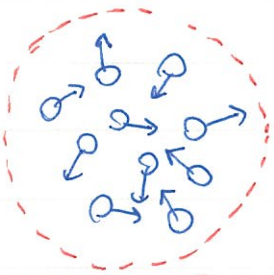


Metal	γ(exp)	γ_0	γ/γ_0
Li	1.63	0.75	2.17
Na	1.38	1.14	1.21
Cu	0.695	0.50	1.39
Ag	0.646	0.65	1.00
	$\mathrm{mJ/mol\cdot K^2}$	$\mathrm{mJ/mol\cdot K^2}$	

The experimental observation shows that it is not so bad to ignore the electron-electron interactions.

It is also interesting to compute the Fermi pressure at $\tau = 0$.

$$U_0 = \frac{3}{5} N \varepsilon_F = \frac{3}{5} N \cdot \frac{\hbar^2}{2m}(3\pi^2 n)^{\frac{2}{3}} = C \cdot V^{-\frac{2}{3}}$$



pressure P.

At $\tau = 0$, $\sigma = 0$ (by the 3rd law).

$$P = -\left(\frac{\partial U}{\partial V}\right)_{\sigma, N} = -C \cdot \left(-\frac{2}{3}\right) V^{-\frac{5}{3}} = \frac{2U_0}{3V}$$

$$\rightarrow \boxed{P = \frac{2U_0}{3V} \neq 0 \text{ even at zero temp!}}$$

The positive pressure tends to expand → What holds the metal stable? The EM interactions ☺

⊘ white dwarf stars

$\rho_\odot \approx 1\ \mathrm{g/cm^3}$

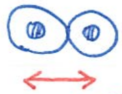
just like water

For a white dwarf star, its mass is roughly the same as the sun but its radius is 1/100 smaller.

$\rho \approx 10^6\ \mathrm{g/cm^3}$ ← Extremely high density! Something like a marble would weight 1 ton!!

white dwarf star

Use H atom to understand the length scale. From the pictures below, it's reasonable to expect that electrons are not attached to individual nuclei.



H_2 $0.74\,\text{Å}$

$0.01\,\text{Å}$

white dwarf star.

$$\boxed{\varepsilon_F = \frac{\hbar^2}{2m}(3\pi^2 n)^{\frac{2}{3}} \approx 10^5\ eV}$$

This is equivalent to $T_F \sim 10^9 K$.

$$P = \frac{2U_0}{3V} = \frac{2}{3V}\cdot\frac{3}{5}N\varepsilon_F = \frac{2}{5}n\cdot\frac{\hbar^2}{2m}(3\pi^2 n)^{\frac{2}{3}} \sim n^{\frac{5}{3}}$$

Compared with electrons in metal, the pressure is larger. What holds the white dwarf star together? Gravity.

② Nuclear matter.

A is mass number.

$$R \cong 10^{-15} m \times A^{\frac{1}{3}}$$

just like a water drop with constant density.

$$\boxed{n \cong \frac{A}{\frac{4}{3}\pi R^3} \approx 10^{39}\ cm^{-3}}$$

It is about 10^8 times larger than that in white dwarf stars.

$$\varepsilon_F = \frac{\hbar^2}{2M}(3\pi^2 n)^{\frac{2}{3}} \approx 10^7 eV = 10 MeV$$

What holds nucleons together? Strong interaction

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