微積分（B）附件

【章節 17.2】

※補充題用

① If R=[-1,3]x [0,2], use a Riemann sum with m=4 and n=2 to estimate the value of

$$\iint_R (y^2 - 2x^2)dxdy.$$

Take the sample points to be the upper left corners of the sub-rectangle Rij.

② Calculate the iterated integral.

ⓐ $\int_1^4 \int_0^2 (x+\sqrt{y})\,dxdy$　　ⓑ $\int_0^1 \int_1^2 \frac{xe^x}{y}\,dydx$

ⓒ $\int_1^4 \int_1^2 \left(\frac{x}{y}+\frac{y}{x}\right)dydx$　　ⓓ $\int_0^1 \int_0^1 xy\sqrt{x^2+y^2}\,dydx$

③ Calculate the double integral.

ⓐ $\iint_R \frac{xy^2}{x^2+1}\,dxdy,\quad R=\{(x,y)\mid 0\le x\le 1,\quad -3\le y\le 3\}$

ⓑ $\iint_R xye^{x^2y}\,dxdy,\ R=[0,1]x[0,2]$

④ The average value of a function of a function f(x,y) over a rectangle R is defined

to by $f_{ave}=\frac{1}{area(R)}\iint_R f(x,y)dxdy.$

Find the average value of f over the given rectangle.

ⓐ$f(x,y)=x^2y$,　R has vertices (-1,0),(-1,5),(1,5),(1,0)

ⓑ$f(x,y)=e^y\sqrt{x+e^y}$,　R=[0,4]x[0,1]

⑤ Let R=[a,b] x [c,d]. Show that if f is continuous on [a,b] and g is continuous on [c,d], then $\iint_R f(x)g(y)dxdy = \left[\int_a^b f(x)dx\right]\left[\int_c^d g(y)dy\right]$