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1. An incompressible fluid flows steadily between two horizontal, infinite parallel plates that are separated by a distance h. By solving the Navier–Stokes equations with suitable boundary conditions, show that the volumetric flowrate per unit width of the plates is

$$q = \frac{h^3 \Delta p}{12\mu l},$$

where μ is the fluid viscosity, and Δp is the pressure drop between two points a distance l apart in the flow direction. This result sometimes is referred to as Hagen–Poiseuille law. To earn full credits, clearly explain your assumptions, mathematical formulation, and algebraic derivations. **(10 %)**

2. Figure P2 shows schematically a *capillary pump* that can be used in lab-on-a-chip systems.

(a) Suppose that the gauge pressure of the ambient air at point E in Fig. P2 is zero, i.e., $p_E = 0$. Also, the meniscus between points D and E is approximately a circular arc. Show that the gauge pressure of the liquid (say, water) at point D is

$$p_D = -\frac{2\sigma}{h}\cos\theta,$$

where σ is the surface tension coefficient, θ the contact angle, and h the separation distance between the two parallel plates shown in Fig. P2. **(5 %)**

(b) For a *wetting* liquid, the contact angle $\theta < \pi/2$, and so $p_D < 0$. Also, for a shallow input reservoir having a relatively large horizontal expanse, we may take $p_C \approx p_B \approx p_A = 0$. In then transpires that $p_C > p_D$, and so the capillary flow will continue as long as there is a channel for the liquid to propagate in. Here, suppose that the fluid is *inviscid*, and calculate the speed of the meniscus advancement dL/dt, $L(t)$ being the position of the meniscus. **(10 %)**

(c) Now, suppose that the liquid is *viscous* and $L(0) = 0$. Calculate $L(t)$. **Hint:** You may use a suitable conservation law and the Hagen–Poiseuille law given in Problem 1 to derive a differential equation for $L(t)$. **(10 %)**

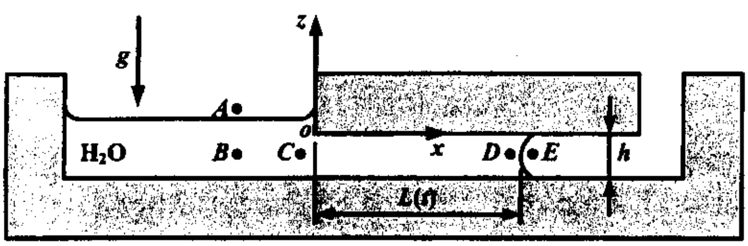

Fig. P2

3. A fan has a bladed rotor of 12 in. outside diameter and 5 in. inside diameter and runs at 1,725 rpm; see Fig. P3. The width of each rotor blade is 1 in. from blade inlet to outlet. The volumetric flowrate is steady at 230 ft^3/min and the absolute velocity of the air at blade inlet, V_1, is purely radial. The blade discharge angle is 30° measured with respect to the tangential direction at the outside diameter of the rotor.

(a) Find the particular blade inlet angle (measured with respect to the tangential direction at the inside diameter of the rotor) that renders the blade tangent to the relative air velocity at the inlet. **(5 %)**

(b) Calculate the power required to run the fan. Take air density $\rho = 2.38 \times 10^{-3}$ slug/ft^3. **(10 %)**

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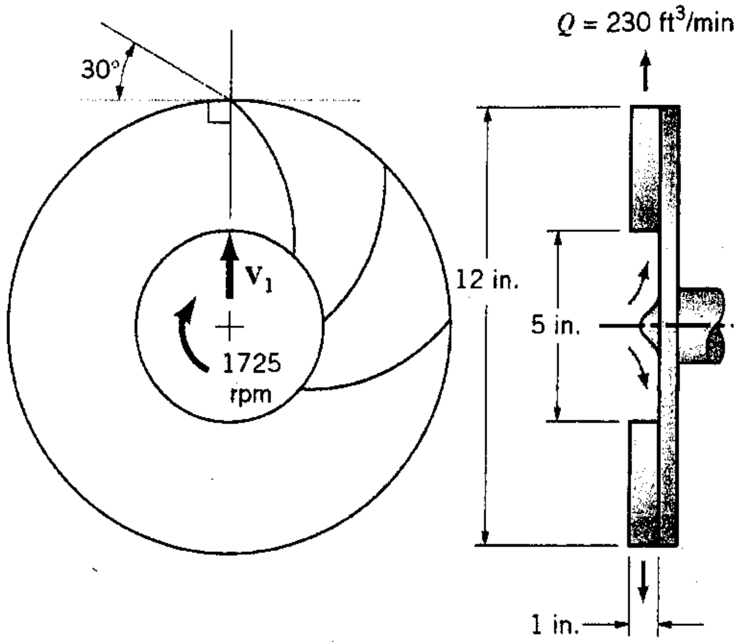

Fig. P3

4. **(10%)** The equations of motion for a jet of hot gas at a absolute temperature T_H moving through a colder fluid at T_0 (absolute) are

$$\frac{D\mathbf{v}}{Dt} = \frac{\nabla p}{\rho_0} + \nu_0 \nabla^2 \mathbf{v} + \mathbf{g}\left(\frac{T_H}{T_0} - 1\right)$$

(a) Taking L and V_0 as the reference length and velocity, respectively, find the non-dimensional parameter which represents the ratio of gravity (buoyancy) to inertial forces acting on a fluid element.

(b) A 1/100 scale model of a stack for discharging hot gases is to be tested in a wind tunnel. The prototype stack is 40 m high and discharges 10^6 m^3/day of a 80°C gas to atmosphere (T_0~20°C). The model gas is limited to 27°C, and also discharges to atmospheric conditions. What is the model discharge rate for dynamic similarity?

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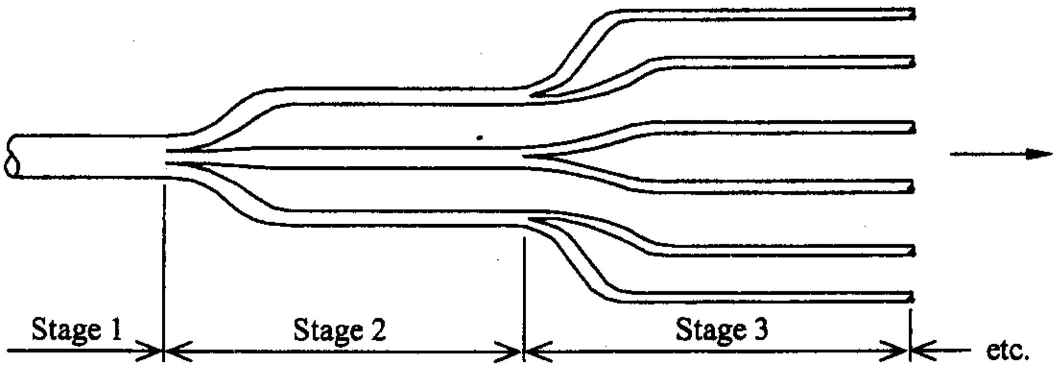
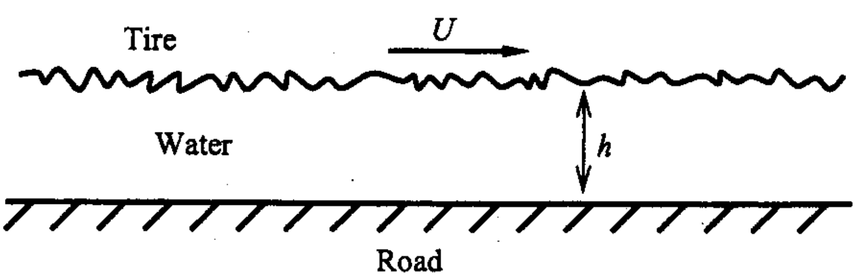
5. **(20%)** Both the mammalian respiration system and the mammalian blood circulation system are networks of tubes in which the flow from one large tube (respectively, the trachea and the aorta) branches into parallel flows in tubes of smaller size. This branching continues through a number of stages as shown in the illustration.



(a) If, for each stage, the number of tubes is denoted by n and the cross-sectional area of each and every tube in that stage is denoted by A_n, find the relation between A_n and n such that the pressure gradient $\partial p / \partial x$ is the same for each stage. How does the average velocity depend on n? (Assume steady, fully developed Poiseuille flow in all tubes.)

(b) If the diameter of the aorta is 3 cm and the diameter of the microcirculation (the smallest tubes) is 8×10^{-6} m, calculate the number of tubes at the microcirculation stage required for the property above to hold. The actual number is a great deal less than this.

6. **(20%)** The rough surface of an automobile tire has a roughness of size, ε. Consider the following Couette flow which models the hydroplaning of the tire on a smooth road:



The speed of the tire is U, the mean liquid film thickness is h, and the kinematic viscosity of the liquid is ν. If the magnitudes of the unsteady turbulent velocities, u' and v', generated by the roughness are both given approximately by $U\varepsilon y / h^2$ where y is the distance above the smooth road, find the ratio of the "effective" dynamic viscosity of the film of liquid to the actual liquid dynamic viscosity. The answer includes U, ε, h and ν.