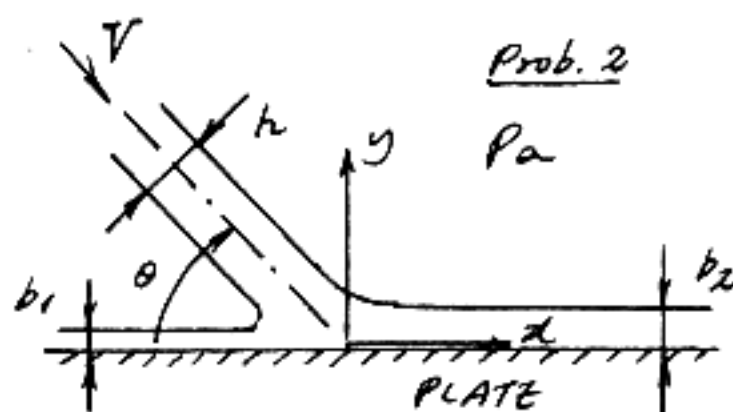
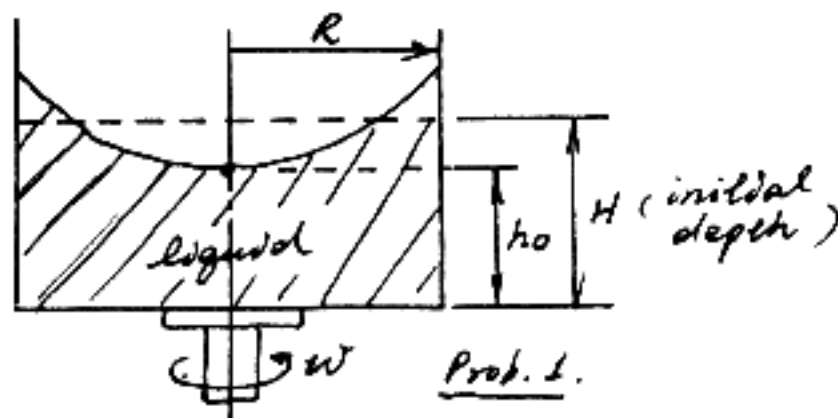


1. It has been suggested that the angular velocity, ω , of a rotating body or shaft can be measured by attaching an open cylinder of liquid, as shown below, and measuring with some type of depth gage the change in the fluid level, $H - h_0$, caused by the rotation of the fluid.

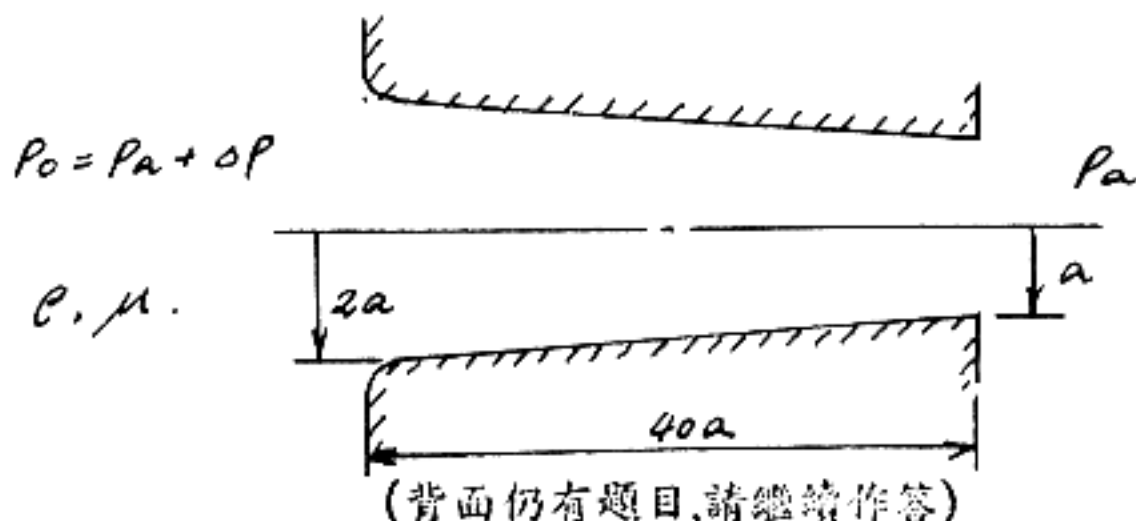
- Determine the relationship between this change in fluid level and the angular velocity. (4%)
- At what angular speed (ω_d) would the fluid depth on the axis of rotation become zero (i.e., $h_0 = 0$)? Calculate the minimum height of the cylinder wall required to avoid fluid spill at $\omega = \omega_d$. (4%)
- Suppose now that the cylinder wall is so tall that we do not have to worry about fluid spill. Sketch the shape of the free surface for $\omega > \omega_d$, and briefly explain how you may calculate the free surface shape. (You do not have to carry out the calculation.) (2%)



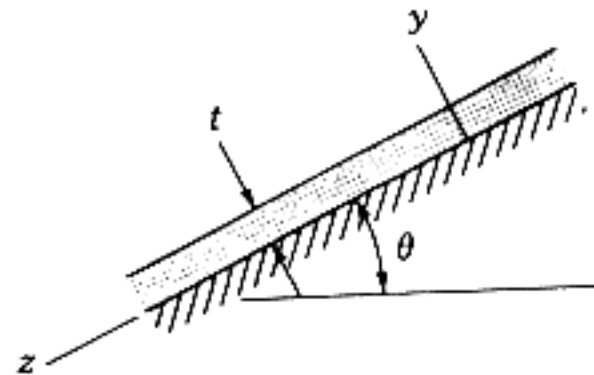
2. A two-dimensional liquid jet (having a density of ρ) of width h impinges at an angle θ onto a smooth, flat plate, and divides into two streams as sketched. The ambient atmosphere is at a constant pressure p_a . Assume that the flow is incompressible and inviscid (i.e., no shear forces can be exerted by the liquid), and neglect gravitational effects. Derive expressions for the x and y components of force, and the clockwise torque, exerted by the flow on the plate (per unit length perpendicular to the figure). Also, obtain expressions for the widths b_1 and b_2 of the two streams parallel to the plate. (15%)

3. Consider a conical nozzle with exit radius a , inlet radius $2a$ and length $40a$. An incompressible fluid of density ρ and viscosity μ flows through the nozzle from a reservoir at pressure $p_0 = p_a + \Delta p$ to the ambient atmosphere at pressure p_a . Gravitational effects are negligible.

- Suppose you were unable to solve this problem theoretically, but set about making experimental measurements on nozzles of various sizes " a " and with various fluids to find the mean exit velocity V_e in terms of the given quantities a , ρ , μ , Δp , and p_a . Based on dimensional analysis, how would you plot your results so as to determine the functional relationships with the greatest economy of experimental effort? (7%)
- Suppose that the flow is **inviscid**. Derive an expression for the mean exit velocity V_e in terms of the given quantities a , ρ , μ , Δp , and p_a . (7%)
- Now consider the opposite limit where the flow is **highly viscous**, so that the viscous forces are much larger than the inertial ones. Derive an expression for the mean exit velocity V_e in terms of the given quantities. (8%)
- Sketch the solutions (b) and (c) on the plot which you suggested in (a), and indicate the regions of the plot where one or the other of these solutions is expected to apply. (3%)



4. Using Navier-Stokes equations, determine the thickness of a film of fluid moving at constant speed and thickness down an infinite inclined wall in terms of volume flow q per unit width. (25%)



5. Air at 27°C and 1 atm flows over a flat plate at a speed of 2 m/s. Assume the plate is heated over its entire length to a temperature of 60°C. Calculate the heat transferred in the first 0.2 m of the plate. The Nusselt number is given as

$$Nux = 0.332P_r^{1/3} Re x^{1/2} \left[1 - \left(\frac{x_0}{x} \right)^{3/4} \right]^{-1/3}$$

(25%)

Properties of Air at Atmospheric Pressure

T, K	ρ kg/m ³	c_p kJ/kg°C	μ kg/ms $\times 10^5$	ν m ² /s $\times 10^4$	k W/m°C	α m ² /s $\times 10^4$	Pr
100	3.6010	1.0266	0.6924	1.923	0.009246	0.02501	0.770
150	2.3675	1.0099	1.0283	4.343	0.013735	0.05745	0.753
200	1.7684	1.0061	1.3289	7.490	0.01809	0.10165	0.739
250	1.4128	1.0053	1.5990	11.31	0.02227	0.15675	0.722
300	1.1774	1.0057	1.8462	15.69	0.02624	0.22160	0.708
350	0.9980	1.0090	2.075	20.76	0.03003	0.2983	0.697
400	0.8826	1.0140	2.286	25.90	0.03365	0.3760	0.689
450	0.7833	1.0207	2.484	31.71	0.03707	0.4222	0.683
500	0.7048	1.0295	2.671	37.90	0.04038	0.5564	0.680
550	0.6423	1.0392	2.848	44.34	0.04360	0.6532	0.680
600	0.5879	1.0551	3.018	51.34	0.04659	0.7512	0.680
650	0.5430	1.0635	3.177	58.51	0.04953	0.8578	0.682
700	0.5030	1.0752	3.332	66.25	0.05230	0.9672	0.684
750	0.4709	1.0856	3.481	73.91	0.05509	1.0774	0.686
800	0.4405	1.0978	3.625	82.29	0.05779	1.1951	0.689
850	0.4149	1.1095	3.765	90.75	0.06028	1.3097	0.692
900	0.3925	1.1212	3.899	99.3	0.06279	1.4271	0.696
950	0.3716	1.1321	4.023	108.2	0.06525	1.5510	0.699
1000	0.3524	1.1417	4.152	117.8	0.06752	1.6779	0.702
1100	0.3204	1.160	4.44	138.6	0.0732	1.969	0.704
1200	0.2947	1.179	4.69	159.1	0.0782	2.251	0.707
1300	0.2707	1.197	4.93	182.1	0.0837	2.583	0.705
1400	0.2515	1.214	5.17	205.5	0.0891	2.920	0.705
1500	0.2355	1.230	5.40	229.1	0.0946	3.262	0.705
1600	0.2211	1.248	5.63	254.5	0.100	3.609	0.705
1700	0.2082	1.267	5.85	280.5	0.105	3.977	0.705
1800	0.1970	1.287	6.07	308.1	0.111	4.379	0.704
1900	0.1858	1.309	6.29	338.5	0.117	4.811	0.704
2000	0.1762	1.338	6.50	369.0	0.124	5.260	0.702
2100	0.1682	1.372	6.72	399.6	0.131	5.715	0.700
2200	0.1602	1.419	6.93	432.6	0.139	6.120	0.707
2300	0.1538	1.482	7.14	464.0	0.149	6.540	0.710
2400	0.1458	1.574	7.35	504.0	0.161	7.020	0.718
2500	0.1394	1.688	7.57	543.5	0.175	7.441	0.730