

1. A fluid flows at a velocity  $V$  through a horizontal pipe of diameter  $D$ . An orifice plate containing a hole of diameter  $d$  is placed in the pipe. It is desired to investigate the pressure drop,  $\Delta p$ , across the plate. Assume that

$$\Delta p = f(D, d, \rho, V)$$

where  $\rho$  is the fluid density. Determine a suitable set of pi terms. (15%)

2. A soft drink with the properties of 10 °C water ( $\rho = 999.7 \text{ kg/m}^3$ ,  $\nu = 1.307 \times 10^{-6}$ ) is sucked through a 4-mm-diameter, 0.25-m-long straw at a rate of 4  $\text{cm}^3/\text{s}$ . Is the flow at the outlet of the straw laminar? Is it fully developed? Explain. (15%)
3. Please define major head loss and minor head loss. (5%)
4. State the conditions under which prototype behavior can be predicted from model tests. (7%)
5. For fully-developed flow in a pipe determine the wall shear stress and the shear stress variation in the flow in terms of the pressure gradient. (8%)

(背面仍有題目,請繼續作答)

6. 回答下列問題 (請完整地將答案寫在答案紙上, 否則不予計分)  
(10%)

(a) The continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

is valid for -----

- (b) If for a flow, a stream function  $\psi$  exists and satisfies the Laplace equation, then the flow is-----  
 (c) In two-dimensional flow the equation of a streamline is given as-----  
 (d) If a stream function  $\psi$  exists it implies that the potential function also exists. True or False  
 (e) In a steady flow, streamline, streakline and pathline can be different from each other. True or False

7. The stream function  $\psi$  for uniform flow past a circular cylinder of radius  $a$  in a polar coordinate is given by

$$\psi = U_{\infty} \left( 1 - \frac{a^2}{r^2} \right) r \sin \theta$$

where  $U_{\infty}$  is the free stream velocity parallel to the horizontal axis. Determine the velocity on the cylinder at  $\theta = 90^\circ$ . (10%)

8. By using velocity triangles for flow upstream (1) and downstream (2) of a turbomachine rotor, prove that the shaft work in per unit mass flowing through the rotor is

$$W_{\text{shaft}} = \frac{V_2^2 - V_1^2 + U_2^2 - U_1^2 + W_1^2 - W_2^2}{2}$$

where  $V$  = absolute flow velocity magnitude,  $W$  = relative flow velocity magnitude, and  $U$  = blade speed (12%)

9. An infinitely long, solid, vertical cylinder of radius  $R$  is located in an infinite mass of an incompressible fluids. Start with the Navier-Stokes equation in the  $\theta$  direction and derive an expression for the velocity distribution for the steady flow case in which the cylinder is rotating about a fixed axis with a constant angular velocity  $\omega$ . You need not consider body forces. Assume that the flow is axisymmetric and the fluid is at rest at infinity. (18%)

Navier-Stokes equation in  $\theta$  direction is given as

$$\rho \left( \frac{\partial v_{\theta}}{\partial t} + v_r \frac{\partial v_{\theta}}{\partial r} + \frac{v_{\theta}}{r} \frac{\partial v_{\theta}}{\partial \theta} + \frac{v_r v_{\theta}}{r} + v_z \frac{\partial v_{\theta}}{\partial z} \right) \\ = -\frac{1}{r} \frac{\partial p}{\partial \theta} + \rho g_{\theta} + \mu \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_{\theta}}{\partial r} \right) - \frac{v_{\theta}}{r^2} + \frac{1}{r^2} \frac{\partial^2 v_{\theta}}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_{\theta}}{\partial z^2} \right]$$