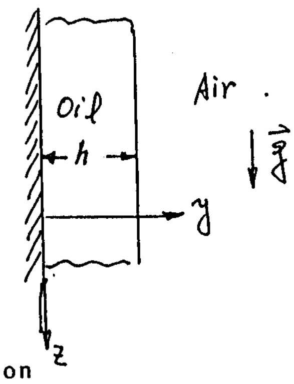
1. Oil is slowly flowing down the flat surface of a large rectangular tank because of gravity, as shown in Figure 1. Its free surface is exposed to air at 1 atmosphere. It has a thickness h. Let air be inviscid, If the flow of oil is steady and two-dimensional,
   (a) determine all stress components in terms of specific weight γ, thickness of oil h, and distance y;
   (b) the velocity field $\vec{V}$;
   (c) show that the flow is irrotational. (15%)



2. Sketch the flow pattern represented by the stream function

$$\psi = Uy\left(1 - \frac{a^2}{x^2 + y^2}\right)$$

where U and a are constants: a velocity and a length respectively. The pressure at $x = \pm\infty$ is $p = p_\infty$. Calculate the pressure along the line $y = 0$. What is the maximum pressure and where does it occur? (10%)

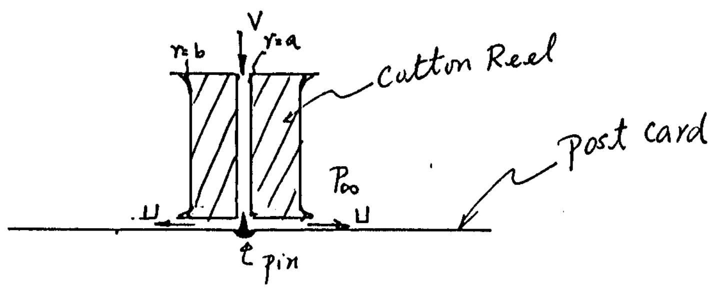
3. In turbulent flow, velocity and pressure fluctuate widely about mean values. Suppose we are using a sensitive instrument to measure velocity at a point in a turbulent flow field. Two velocities can be identified: the mean velocity $\overline{V}_i$ which is an average or mean value, and the instantaneous velocity V_i', which fluctuates randomly about the mean velocity.
   (a) Give the Reynolds stresses in terms of velocity fluctuations.
   (b) Are the turbulent stresses (Reynolds stresses) dependent on viscosity? If yes, explain the relations; if no, where do they come from in the equations of motion?
   (c) Define the mixing length, ℓ, and what is its physical description?
   (e) Explain the Reynolds stress τ_{xy} which can be expressed as

$$\tau_{xy} = -c\ell^2 \left|\frac{d\bar{u}}{dy}\right| \frac{d\bar{u}}{dy}$$

   (f) Why is it essential to express the Reynolds stress τ_{xy} in terms of the mixing length ℓ and the mean-velocity gradient $d\bar{u}/dy$? (25%)

4. (a) What is the so-called "Magnus effect"? (4%)
   (b) For viscous fluid flow the solid boundary is the only place where vorticity can originate. True or False. Why? (4%)
   (c) If, for turbulent flow through a pipe, it is observed that the pressure gradient is proportional to $Q^{7/4}$ where Q is the volumetric flow rate, predict how the pressure gradient would vary with the viscosity and density of the fluid for fixed Q. (7%)

5. Consider an apparatus as shown in the sketch consisting of a cotton reel, a post card, and a drawing pin. The pin is used to keep the center of the card at the center of the hole in the reel. If you blow hard through the hole, the card stays up by itself, despite its weight pulling it down and the downward jet of air impinging on it. On the basis of fluid mechanics, try to give a comprehensive analytical explanation for the above-described observation. (15%)



6. Consider flow in an underground channel between two slightly porous walls. Suppose water percolates in through one wall and out through the other at the same speed u(taken to be a constant), while the stream flows along the channel at the speed U(y). The velocity can be taken as $\vec{V} = U(y)\ \vec{i} + u\ \vec{j}$.
   (a) Derive governing differential equaitons and boundary conditions for the flow problem.
   (b) Determine U(y). (20%)