Part A

1. What is the (mathematical) relationship between circulation & vorticity? 5%

   Prove the existence of a velocity potential in an irrotational flow from the concept of circulation. 10%

2. State briefly and carefully what are the functional relationships for (1) the law of the wall, and (2) the viscous sublayer for turbulent flow in pipes. 8%

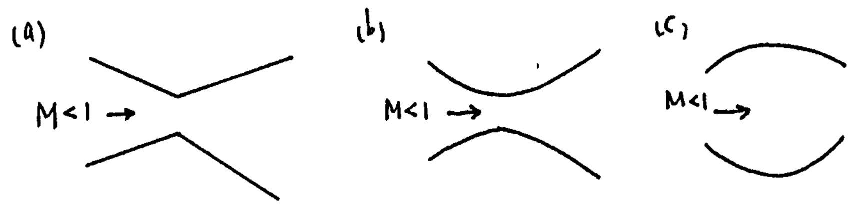
   Prove that the relation in Part (2) is indeed the relation for laminar flow. 7%

3. Consider laminar, steady flow of an incompressible fluid past an infinite flat plate. Now, let the fluid be withdrawn by a constant suction velocity 'V_0' through the plate, such that the boundary layer does not grow with distance along the plate (which is the x direction) with the condition $\partial u/\partial x = 0$. Show how to determine the (transverse) flow velocity distribution of the flowfield. 20%

Part (B)

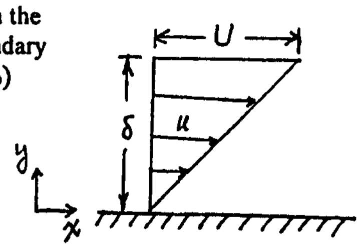
1. Is it true that the total pressure can be computed by $P_0 = p + \frac{1}{2}\rho V^2$ for both compressible and incompressible flow. Explain your reasons. (7%)

2. In the following three nozzle geometries, which one(s) can be accelerated from subsonic speed at inlet to supersonic speed at exit? Explain your reasons. (7%)



3. Can a vortex contain zero vorticity? Give an example to explain your answer.(6%)

4. (a).Derive the momentum-integral equation governing the boundary layer on a semi-infinite flat plate aligned with a uniform free stream at infinity, U. (15%)
(b).Obtain an approximate solution for the flow from the momentum-integral of Part (a) by assuming the boundary layer can be represented by the linear profile (15%)



$$\frac{u}{U} = \eta \qquad \text{where } \eta = \frac{y}{\delta(x)}$$

Calculate δ_1 (displacement thickness), and τ_w (wall shear stress), then compare with the exact Blasius results $(\upsilon = \mu / \rho)$

$$\delta_1 = 1.721\sqrt{\frac{\upsilon x}{U}}, \qquad \tau_w = 0.332\mu U\sqrt{\frac{U}{\upsilon x}}$$