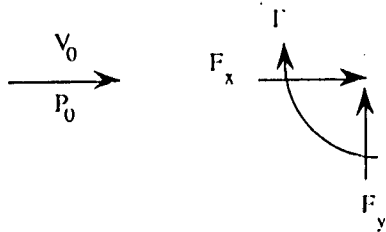


1. Neglecting viscosity, show that lines of constant pressure are perpendicular to the combined force field of a flow field. 15%

2.



Given a vortex of strength  $\Gamma$  located at the origin in a uniform flow of velocity  $V_0$ . Show steps how to compute  $F_x$  and  $F_y$  that are necessary to maintain the vortex in equilibrium by means of the momentum theorem applied to a control volume enclosing this vortex. 20%

3. Define the time-averaged and fluctuating velocities of a turbulent flow. State briefly the key ideas behind this definition. Show that, for incompressible flow, the time-averaged velocity components and the fluctuating components each satisfy the incompressible equation of continuity. 15%

4. (a) What do  $\frac{\partial b}{\partial t}$ ,  $\frac{db}{dt}$  and  $\frac{Db}{Dt}$  mean physically?  $b$  is a property of fluid. 9%

(b) For two problems of fluid flow, their governing equation can be written as the following dimensionless equation,

$$\nabla \cdot \vec{V} = 0$$

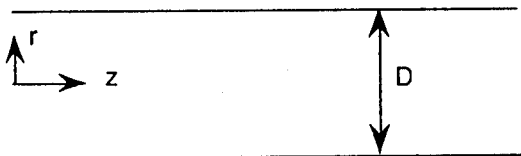
$$\text{Re} \frac{D\vec{V}}{Dt} = -\nabla P + \nabla^2 \vec{V}$$

where  $\text{Re}$  is Reynolds number. In what conditions are these two problem similar? When the two problems are similar, the solution of one problem can be applied to the other one. 7%

5. Consider a fluid flow inside a tube (fully-developed).

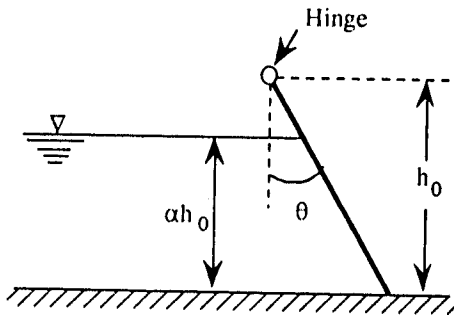
(a) What's the definition or physical meaning of "fully-developed" in the tube flow? 4%

(b) Derive the velocity distribution of the fully-developed flow. 10%



6. (a) A body of water is held in place by a hinged gate as shown in the following figure. The water level is below the elevation  $h_0$  of the hinge. The gate has a weight  $w$  per unit length (normal to the page) and is of uniform thickness. The gate is held closed solely due to its

weight. What is the value of  $w$  at which the gate opens if the water level reaches a height  $\alpha h_0$ , where  $\alpha$  is between 0 and 1? 10%



- (b) A water tank has an orifice at the bottom of the tank. The cross-sectional area of the jet at the point where it leaves the orifice (at  $x=0$ ) is  $A_0$ . The height of the water in the tank is  $h$ , and this height is kept constant by continuously refilling the tank. The tank cross section is much larger than the orifice area. Neglecting friction, surface tension, and so on, find the cross-sectional area  $A$  of the jet as a function of  $x$ . 10%

