※ 考生請注意：本試題 ☑可 ☐不可 使用計算機

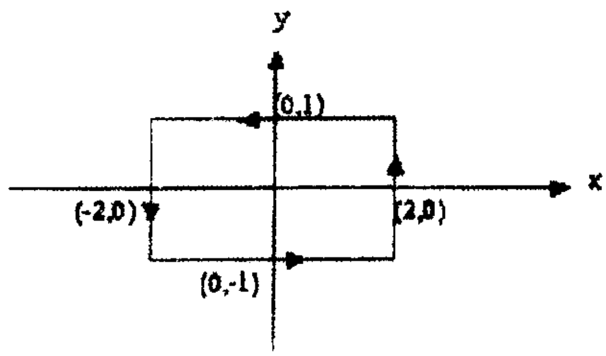
1. Given a stream function $\psi = x^2 y + xy - \frac{1}{3} y^3$

(a) Find out the velocity field $\vec{V} = u(x,y)\vec{i} + v(x,y)\vec{j}$ (4%)

(b) Show that this velocity field represents a possible incompressible flow. (4%)

(c) Determine the velocity field whether a rotational flow or irrotational flow. (4%)

(d) Calculate the circulation $\Gamma = \oint_c \vec{V} \bullet d\vec{s}$ around the square enclosed by x=±2 and y=±1 shown in the figure below (4%)



(e) Calculate the acceleration of a fluid element in the velocity field as it pass through the point (1,1). (4%)

2.The pressure drop, △P, for steady, incompressible viscous flow through a straight horizontal pipe depends on the pipe length L, the average velocity V, the fluid viscosity, μ, the pipe diameter D, the fluid density ρ and the average "roughness" height e. Apply dimensional analysis and determine a set of nondimensionless groups that can used to correlate data. (20%)

3. Figure below shows a viscous, incompressible, Newtonian liquid flow in steady, laminar flows down a vertical wall. The thickness, δ, of the liquid film is constant. Since the liquid free surface is exposed to atmospheric pressure, there is no pressure gradient. For this gravity driven flow, simplify the governing equations to model this flow field and answer the following questions:

(a) Write down and simply the continuity and Navier-Stoke equations with the assumption you use. (4%)

(b) Find out the velocity profile u(y). (4%)

(c) Find out the maximum velocity U_{max}? (4%)

(d) Find out the volume flow rate per unit depth (Q/l) of surface normal to diagram (4%)

(e) Average velocity V_{ave} of the flow field. (4%)

(背面仍有題目,請繼續作答)

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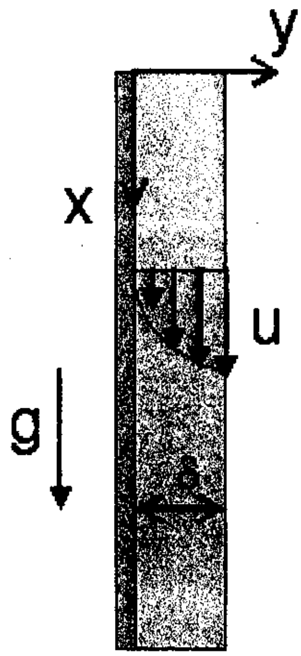
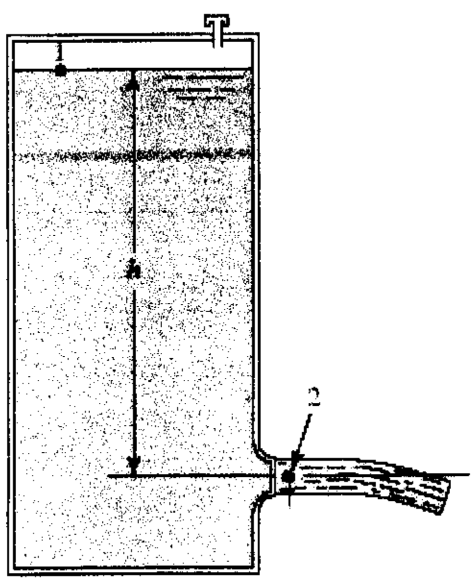


4. For a water tank shown in figure below,

(a) Assuming that there are no energy losses in the system, compute the velocity V_2 of flow from the nozzle (diameter d=25mm) for a fluid depth h of 3.00 m. (10%)

(b) If the minor head loss for nozzle equals $h_l = 0.04\frac{V_2^2}{2}$, solve the velocity V_2 of flow from the nozzle (diameter d=25mm) for a fluid depth h of 3.00 m. (10%)



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5. As shown in Figure below, consider a very small spherical ball with radius R and density ρ_b is allowed to fall freely from a point A in a fluid with a density of ρ_f. The initial velocity of the ball at point A equals zero. The small ball moving in fluids is known to experience buoyant force F_B, gravity force F_G, and drag force F_D. The drag force F_D is proportional to speed.V and can be formulated as $F_D=6\pi\mu VR$, where μ is viscosity of fluid, V is the ball speed and R is the ball radius. The ball reaches its terminal speed V_t as it passes through point B and takes time T (second) falling from point B to point C. The distance between point B and point C is L. Please answer the following questions:

(a) Sketch the free body diagram of the ball and evaluate buoyant force F_B, and gravity force F_G, (4%)

(b) Write down governing equation for the motion of the ball. (4%)

(c) Solve the governing equation and find out the velocity as function of time V(t). (4 %)

(d) Determine the terminal speed V_t in terms of ρ_b, ρ_f, R, μ, and g. (4%)

(e) The falling ball experiment can be used to measure viscosity of a fluid. Determine the viscosity μ of fluid in terms of ρ_b, ρ_f, R, μ, L, T and g. (4%)