編號：F 209　系所：奈米科技暨微系統工程研究所　　科目：流體力學

本試題是否可以使用計算機：□可使用，☑不可使用　（請命題老師勾選）

1. Consider the flow field given by stream function, $\psi = ax^2 - ay^2$, where a is constant.
   (a) Show that the flow is irrotational. (10%)
   (b) Determine the velocity potential for this flow. (10%)

2. A shaft of radius r_i is held concentrically in a cylindrical case of radius r_0. The shaft is kept stationary and the case is moved at a constant velocity V_0 (see Fig. 1). There is no pressure gradient and the flow is laminar and incompressible.
   (a) Determine the velocity distribution of this flow with viscosity μ (10%)
   (b) What force (per unit length) is needed to move the outer cylinder? (10%)

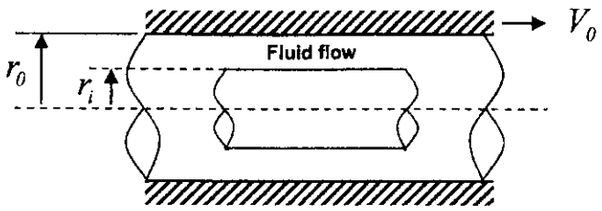

Fig. 1

3. A thin film of liquid falls down a vertical flat plate (see Fig. 2). The viscous shear force and the gravity force acting on the film are assumed in balance. This implies that the film flows steadily with a constant thickness δ.
   (a) Please simplify the following boundary layer equation for this situation. (10%)

$$u\frac{\partial u}{\partial x} + \mathrm{v}\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + g + \nu\frac{\partial^2 u}{\partial y^2}$$

   (b) Write the boundary conditions for the equation by neglecting the density and drag force from the surrounding air. (10%)
   (c) Solve the velocity distribution in the film in terms of δ. (10%)

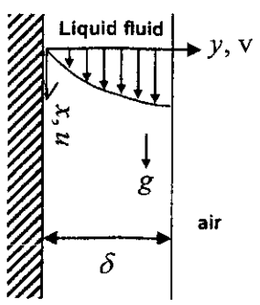

Fig. 2

（背面仍有題目，請繼續作答）

4. **Consider two-dimensional laminar boundary-layer flow along a flat plate (see Fig. 3). Assume the velocity profile in the boundary layer is sinusoidal,**

$$\frac{u}{U} = \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$$

Find expressions for:

(a) the rate of growth of δ as a function of x. (10%)

(b) the displacement thickness, δ^*, as a function of x. (10%)

(c) the total friction force on a plate of length L and width b. (10%)

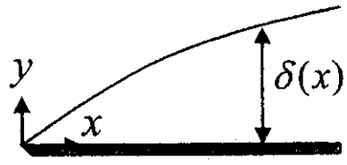

Fig. 3