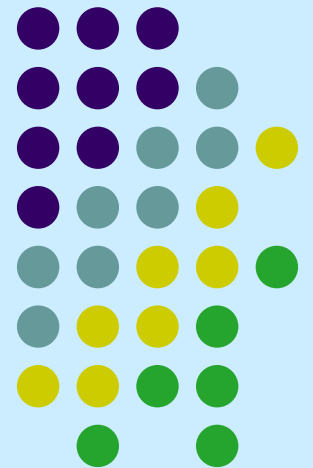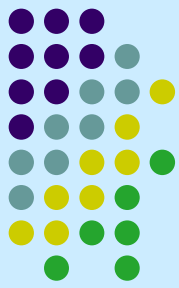# Chapter 9
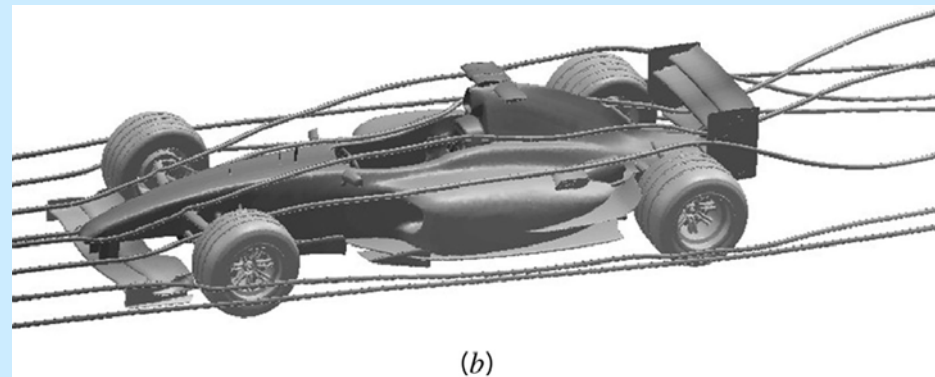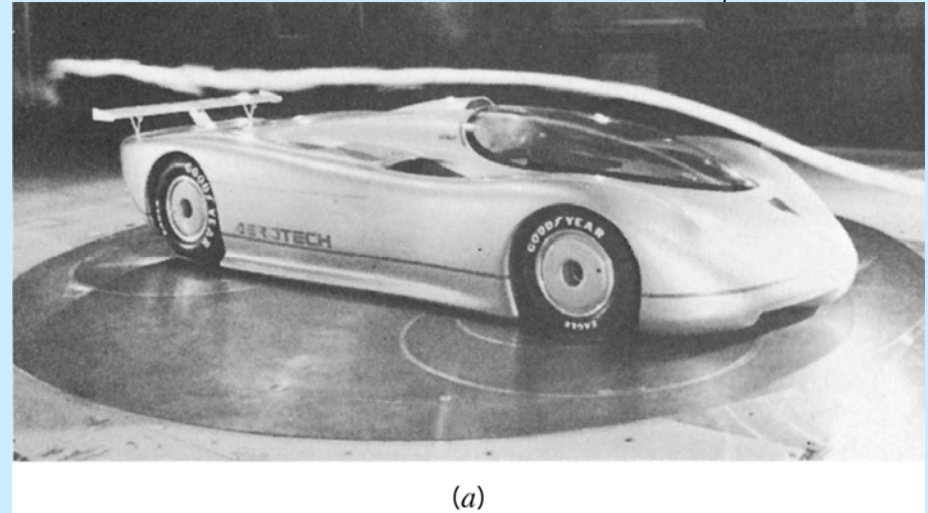
Flow over Immersed Bodies

- We consider flows over bodies that are immersed in a fluid and the flows are termed external flows.

- We are interested in the fluid force (lift and drag) over the bodies. For example, correct design of cars, trucks, ships, and airplanes, etc. can *greatly decrease the fuel consumption and improve the handling characteristics* of the vehicle.

(a)

(b)

9.1 General External Flow Characteristics
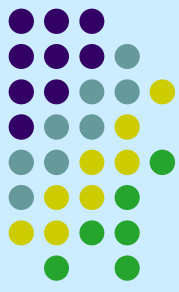
- A body immersed in a moving fluid experiences a resultant force due to the interaction between the body and the fluid surrounding it.
- Object and flow relation:
  stationary air with moving object or
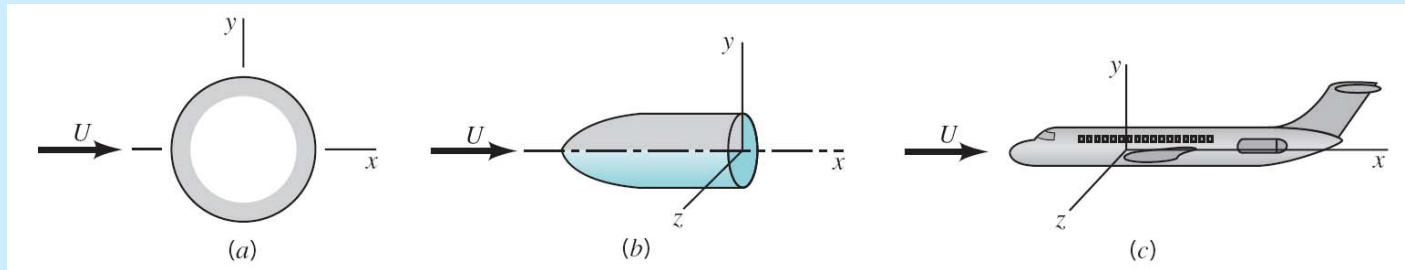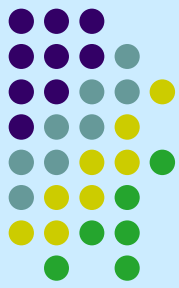 flowing air with stationary object
- Flow classification:

Figure 9.2
Flow classification: (*a*) two-dimensional, (*b*) axisymmetric, (*c*) three-dimensional.

- Body shape:
  Streamlined bodies or blunt bodies

9.1.1 Lift and Drag Concepts
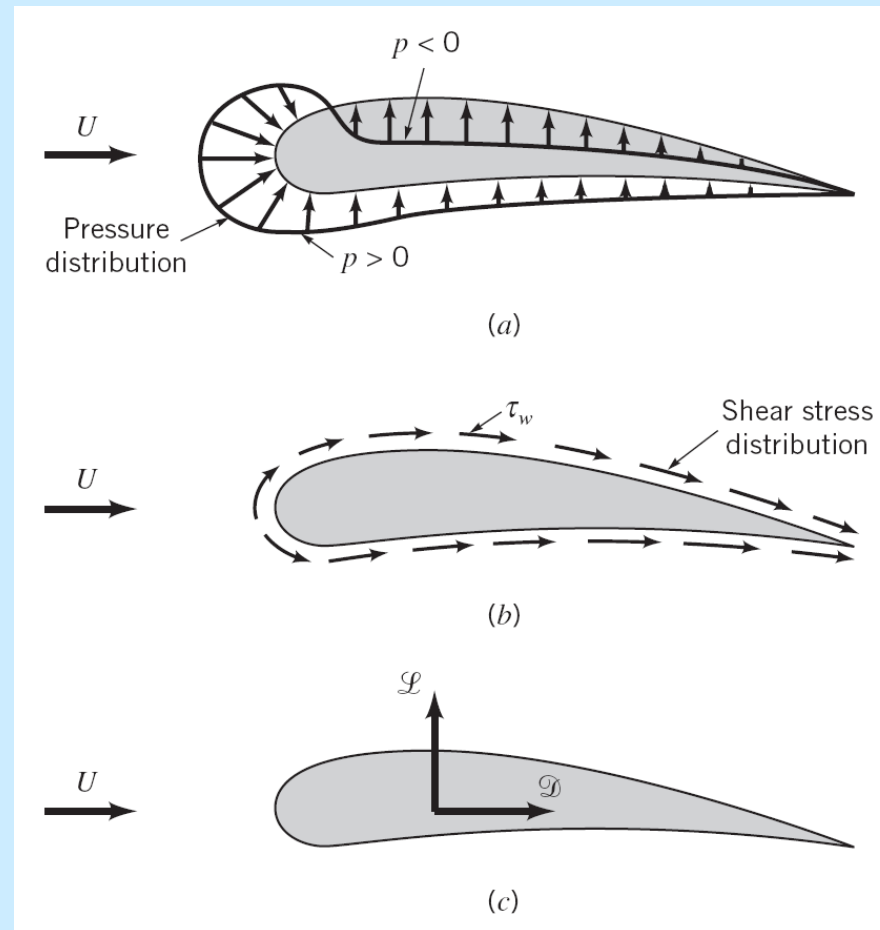
- When any body moves through a fluid, an interaction between the body and the fluid occurs

- This can be described in terms of the stresses-wall shear stresses due to viscous effect and normal stresses due to the pressure P
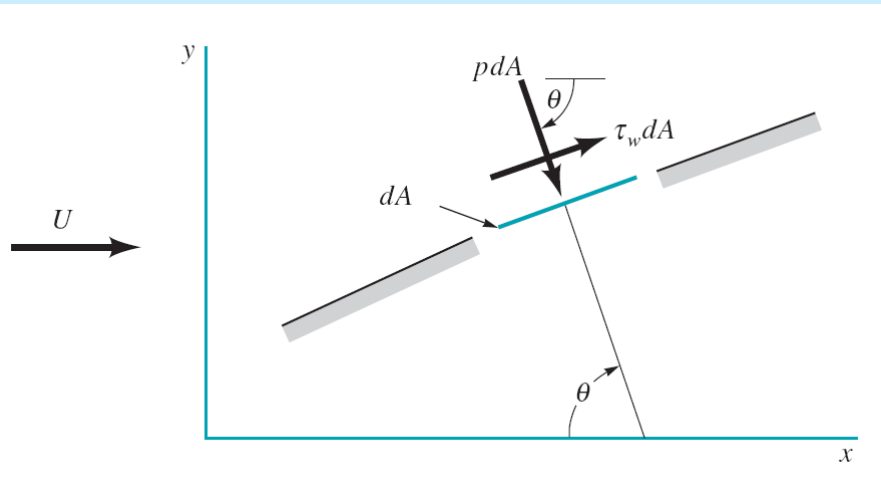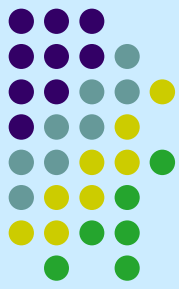
- **Drag**, D

  the resultant force in the direction of the upstream velocity

- **Lift**, L

  the resultant force normal to the upstream velocity

Resultant force--lift and drag

$$dF_x = (pdA)\cos\theta + (\tau_w dA)\sin\theta$$

and

$$dF_y = -(pdA)\sin\theta + (\tau_w dA)\cos\theta$$

The net x and y components of the force on the object are,

$$D = \int dF_x = \int p\cos dA + \int \tau_w \sin\theta dA$$

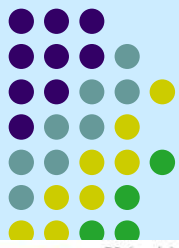$$L = \int dF_y = -\int p\sin\theta dA + \int \tau_w \cos dA$$

● Nondimensional lift coeff. C_L and drag coeff. C_D

$$C_L = \frac{L}{\frac{1}{2}\rho U^2 A}, \quad C_D = \frac{D}{\frac{1}{2}\rho U^2 A}$$

A: charateristic area, frontal area or planform area?

Note: When C_D or $C_L{=}1$, then D or L equals the dynamic pressure on A.

Velocity and pressure distribution around an air foil

At a 7° angle of attack the boundary layer separates from the upper surface of this airfoil, whereas the boundary layer remains attached to the lower surface. Low velocities (top panel) are evident over much of the upper surface. The corresponding pressure distribution (bottom panel) is expressed using the pressure coefficient C_p. A C_p value of zero corresponds to the ambient pressure, with positive pressure indicated by $C_p > 0$ and negative pressures (vacuum) by $C_p < 0$. From the pressure distribution, we see that very little lift (net upward-directed pressure force) is generated at this condition.
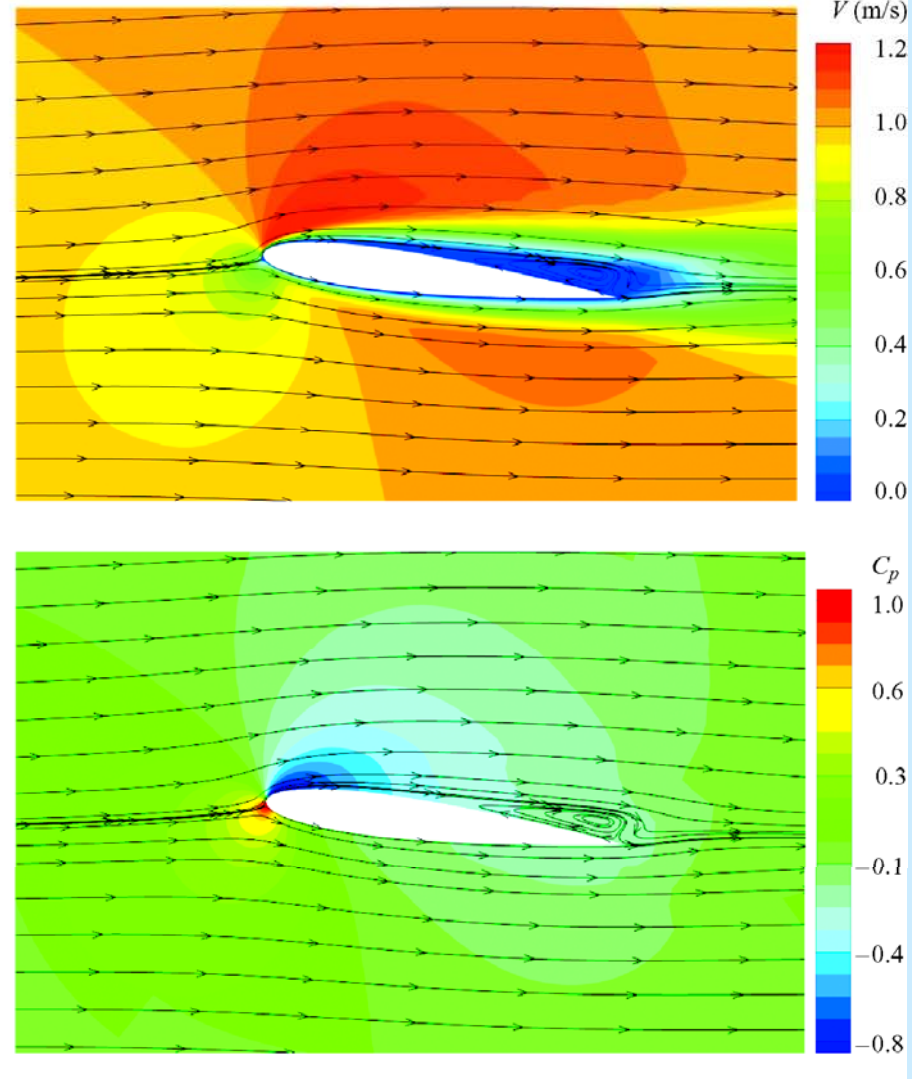
(From S.R. Turns, Thermal-Fluid Sciences, Cambridge Univ. Press, 2006)
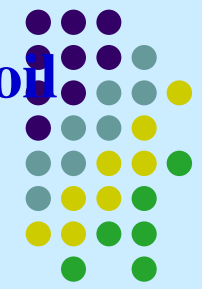
# Drag and lift due to *pressure* and *friction*: example of an air foil
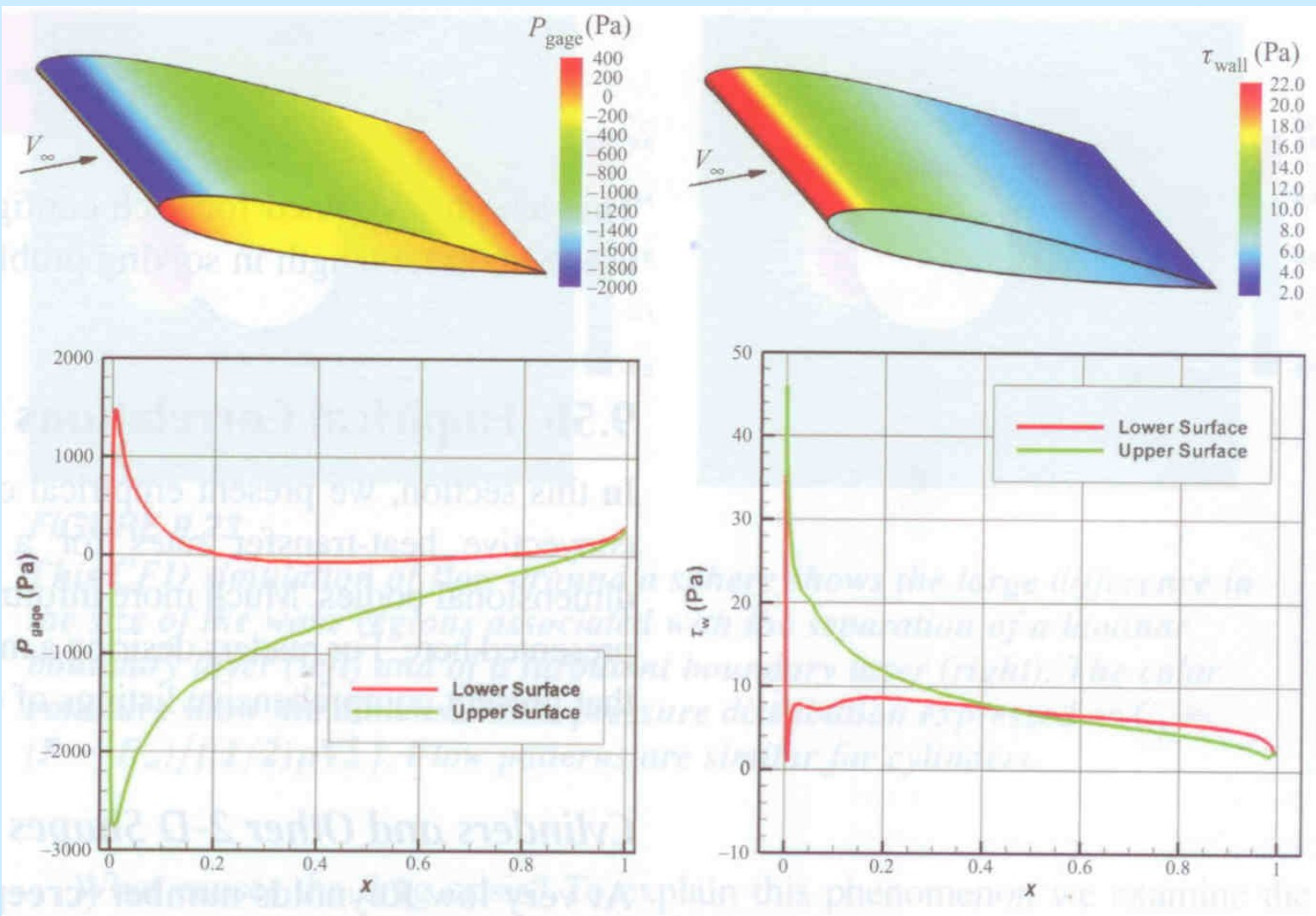
CFD results for $Re = 10^6$ show that <u>pressure forces dominate the lift and drag for an airfoil</u> (NACA 0012) at a 5° angle of attack. Here <u>wall shear stresses (right) are about two orders of magnitude less than surface pressures</u> (left).

(From S.R. Turns, Thermal-Fluid Sciences, Cambridge Univ. Press, 2006)

9.1.2 Characteristics of Flow Past an Object
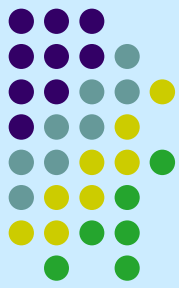
- For typical external flows the most important of parameters are

the **Reynolds number** $\qquad \text{Re} = \dfrac{\rho U l}{\mu}$

the **Mach number** $\qquad Ma = U/c$

the **Froude number**, for flow with a free surface

$$Fr = U / \sqrt{gl}$$

- Flows with Re>100 are dominated by inertia effects, whereas flows with Re<1 are dominated by viscous effects.

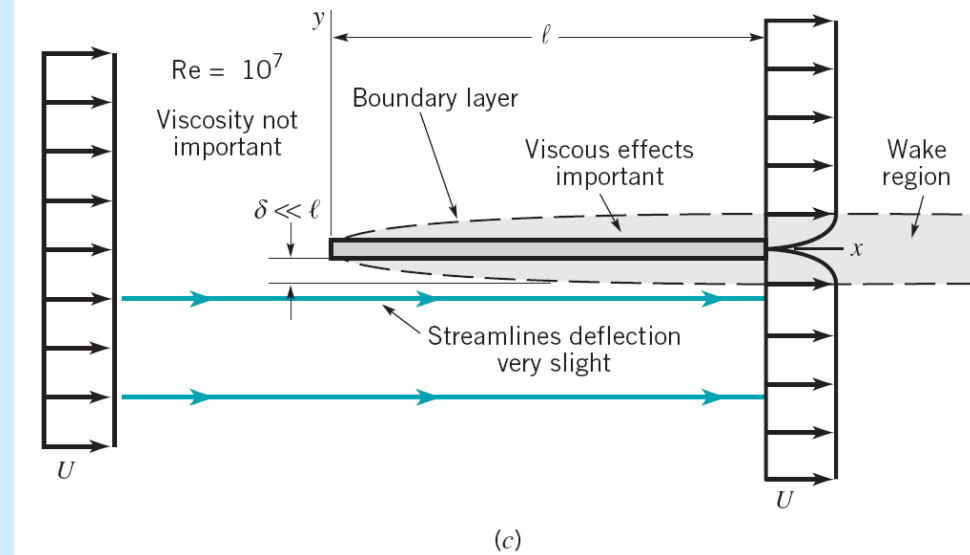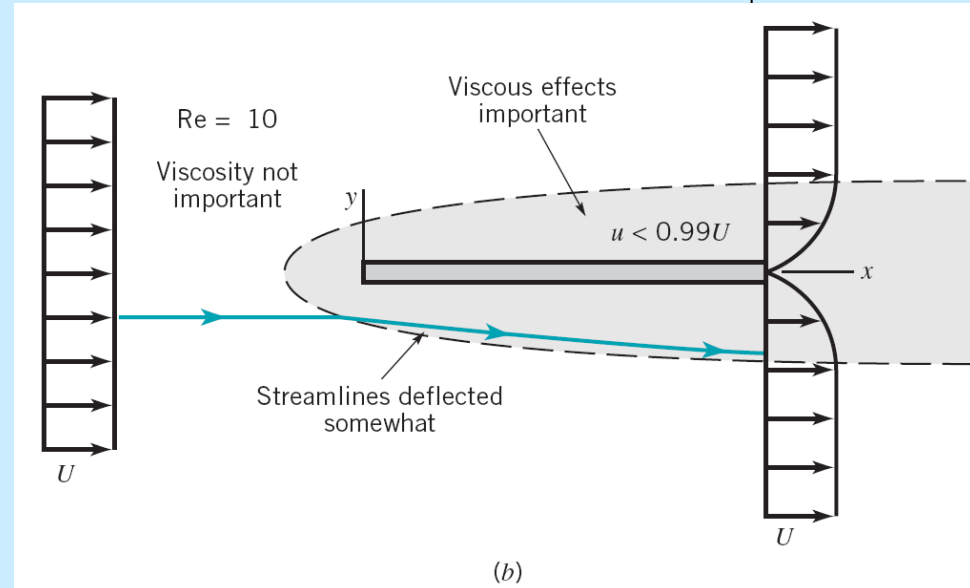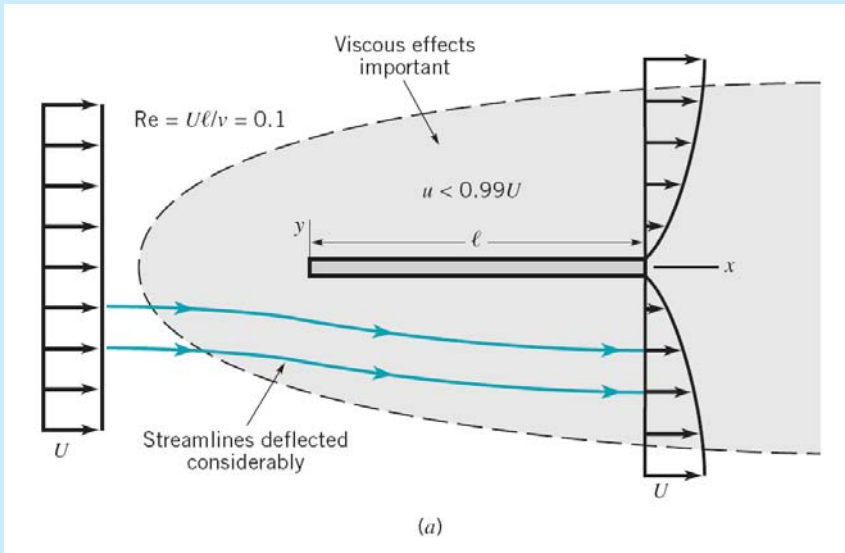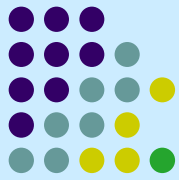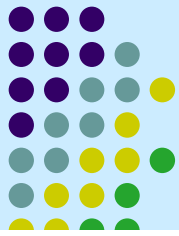# Flow past a flat plate (streamlined body)

Figure 9.5
Character of the steady, viscous flow past a flat plate parallel to the upstream velocity: (*a*) low Reynolds number flow, (*b*) moderate Reynolds number flow, (*c*) large Reynolds number flow.

Steady flow past a circular cylinder (blunt body)

- The velocity gradients within the boundary layer and wake regions are much larger than those in the remainder of the flow fluid.

- Therefore, viscous effects are confined to the boundary layer and wake regions.

U

Viscous forces
important throughout
Re = UD/ν = 0.1

D

x

(a)

Viscosity not
important

Separation
location
+

Re = 50

Separation bubble

Viscous
effects
important

x

(b)

Viscosity not
important

Boundary layer separation

Boundary layer

$\delta \ll D$

D

Re = 10^5

Viscous effects
important

Wake
region

Separated region

x

(c)
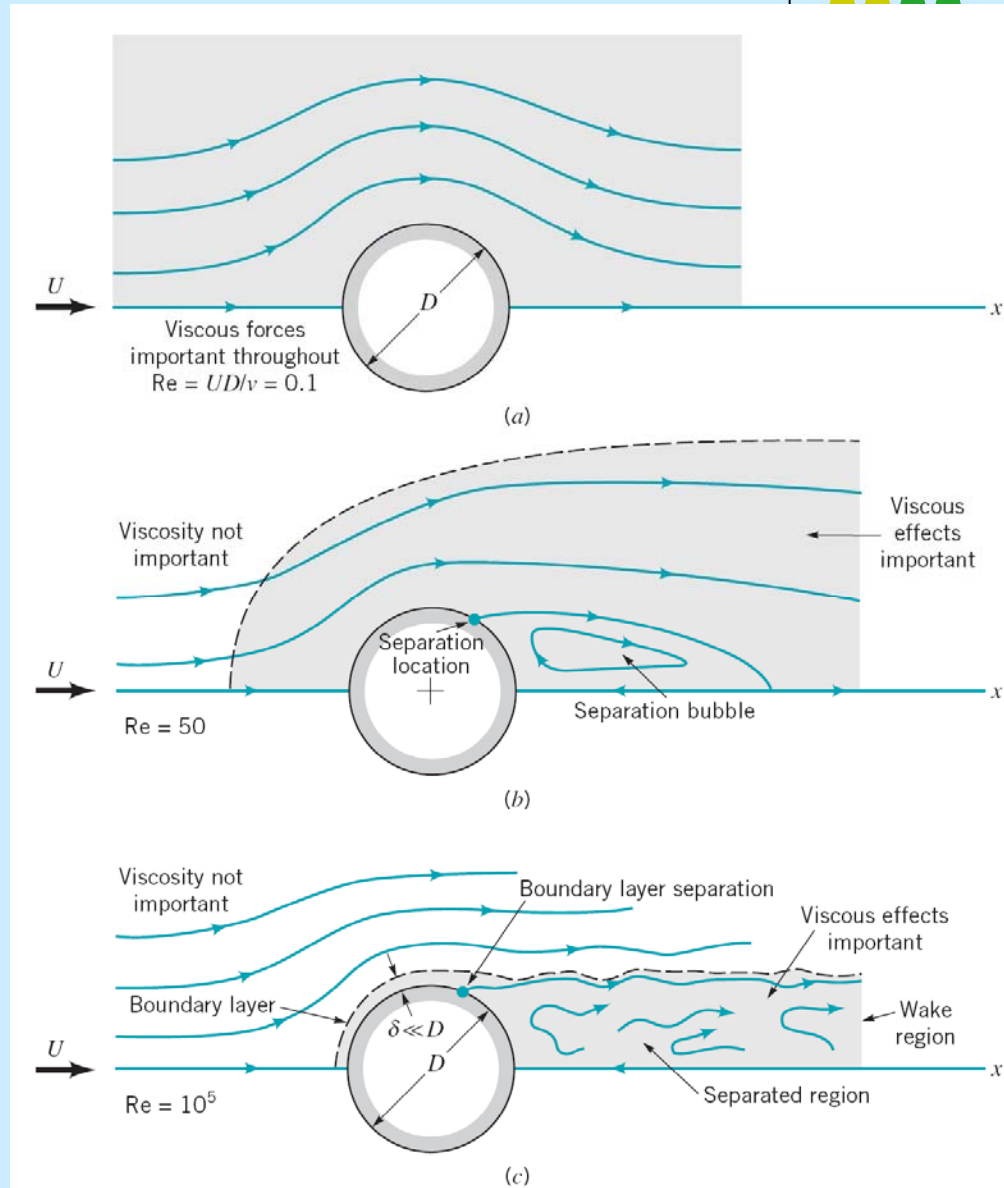
9.2 Boundary Layer Characteristics
9.2.1 Boundary Layer Structure and Thickness on a Flat Plate
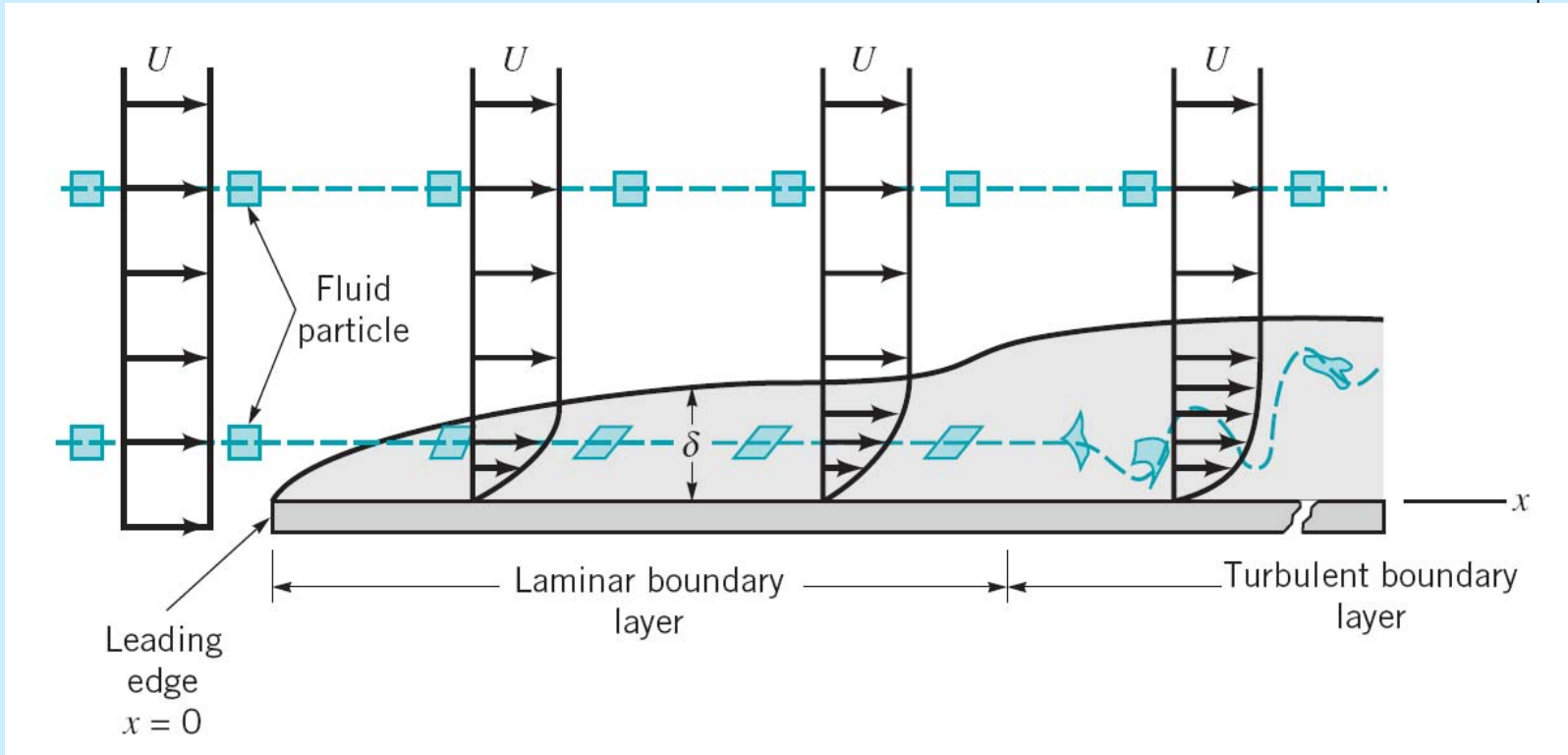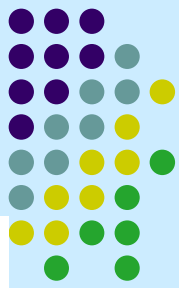

Figure 9.7
Distortion of a fluid particle as it flows within the boundary layer.

- Boundary layer thickness, δ

$$\delta = y \text{ where } u = 0.99U$$

Boundary Layer Structure and Thickness on a Flat Plate
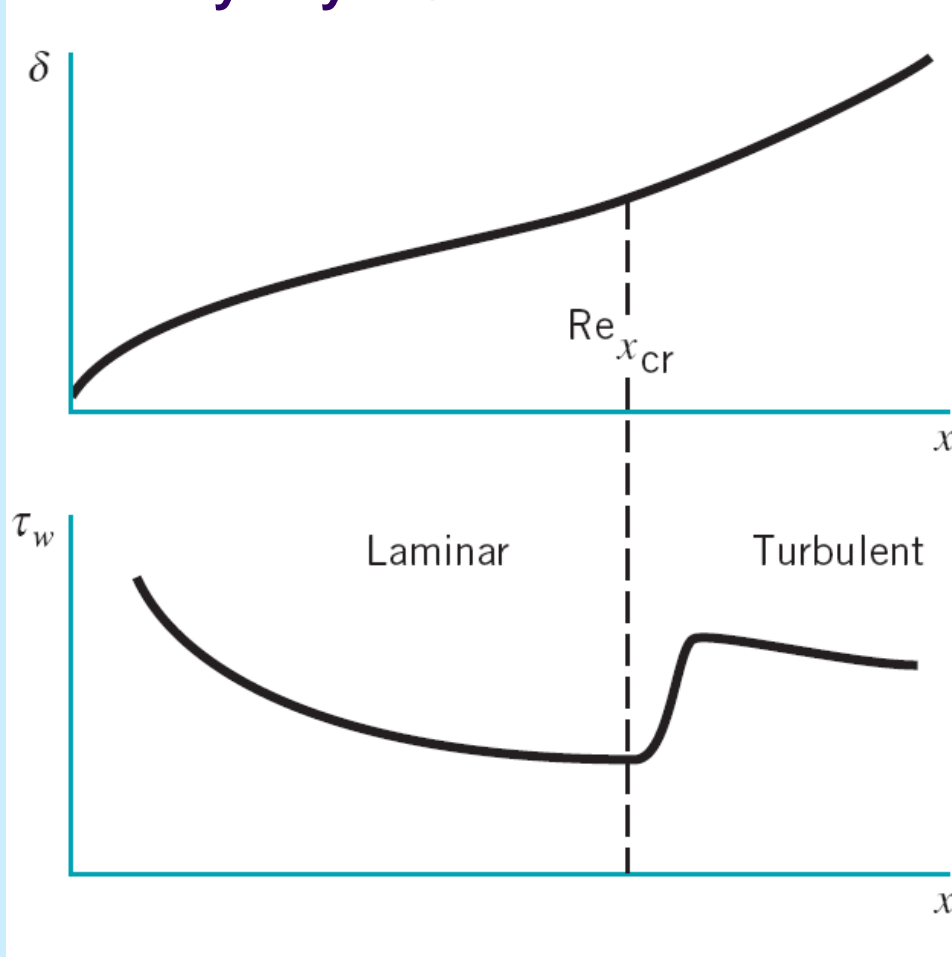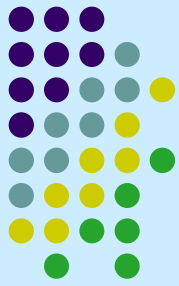
Transition occurs at

$$\mathrm{Re}_{xcr} = \frac{\rho U x}{\mu} = 2\times10^5 \sim 3\times10^6$$

depending on surface roughness and amount of turbulence in upstream flow

Figure 9.9

Typical characteristics of boundary layer thickness and wall shear stress for laminar and turbulent boundary layers.

V9.3 Laminar boundary layer

V9.4 Laminar/turbulent transition

Boundary layer displacement thickness

$$\delta^* b U = \int_0^{\infty \text{ or } \delta} (U - u) b \, dy$$

$$\delta^* = \int_0^{\infty} \left(1 - \frac{u}{U}\right) dy \qquad (9.3)$$
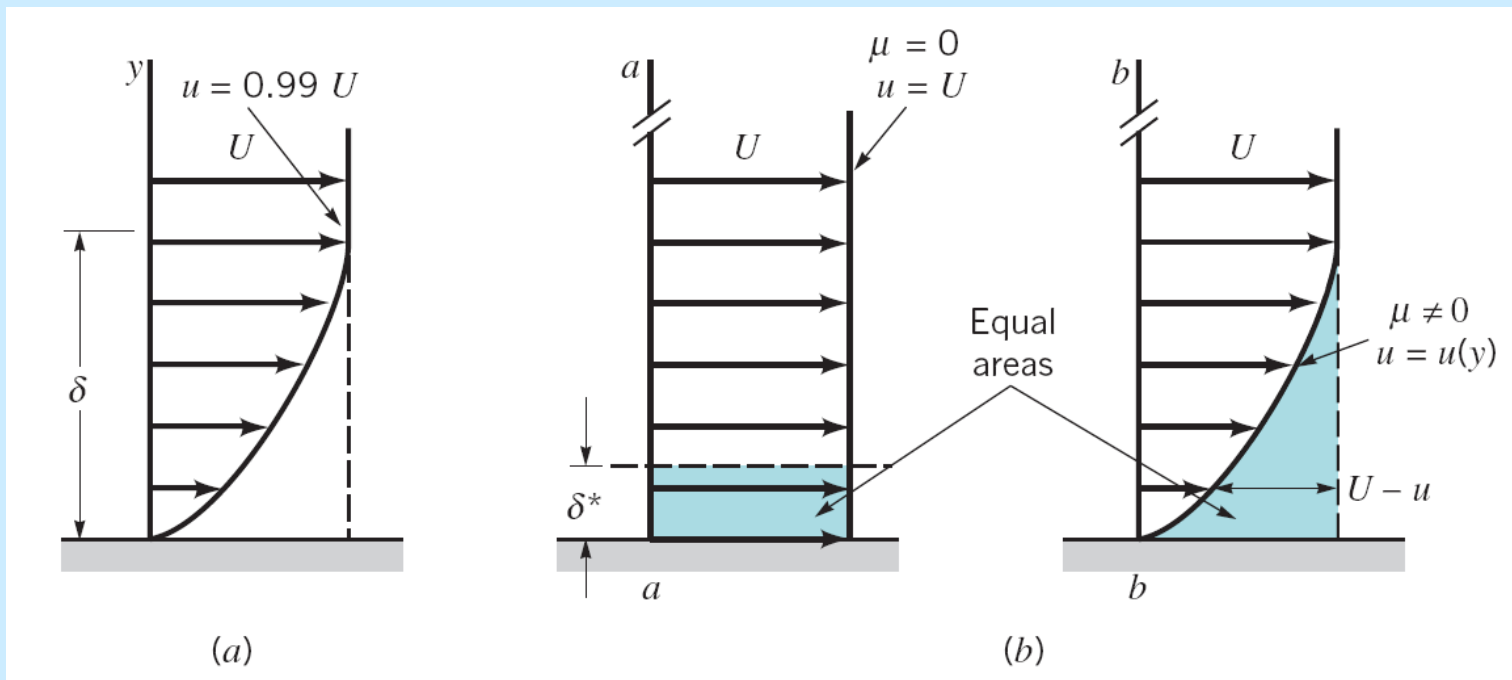

Figure 9.8 (p. 472)
Boundary layer thickness: (a) standard boundary layer thickness, (b) boundary layer displacement thickness.

Boundary layer displacement thickness

- Represents the amount that the thickness of the body must be increased so that the fictitious uniform inviscid flow has the same mass flow rate properties as the actual viscous flow.

- It represents the outward displacement of the streamlines caused by the viscous effects on the plate.

(a)   (b)
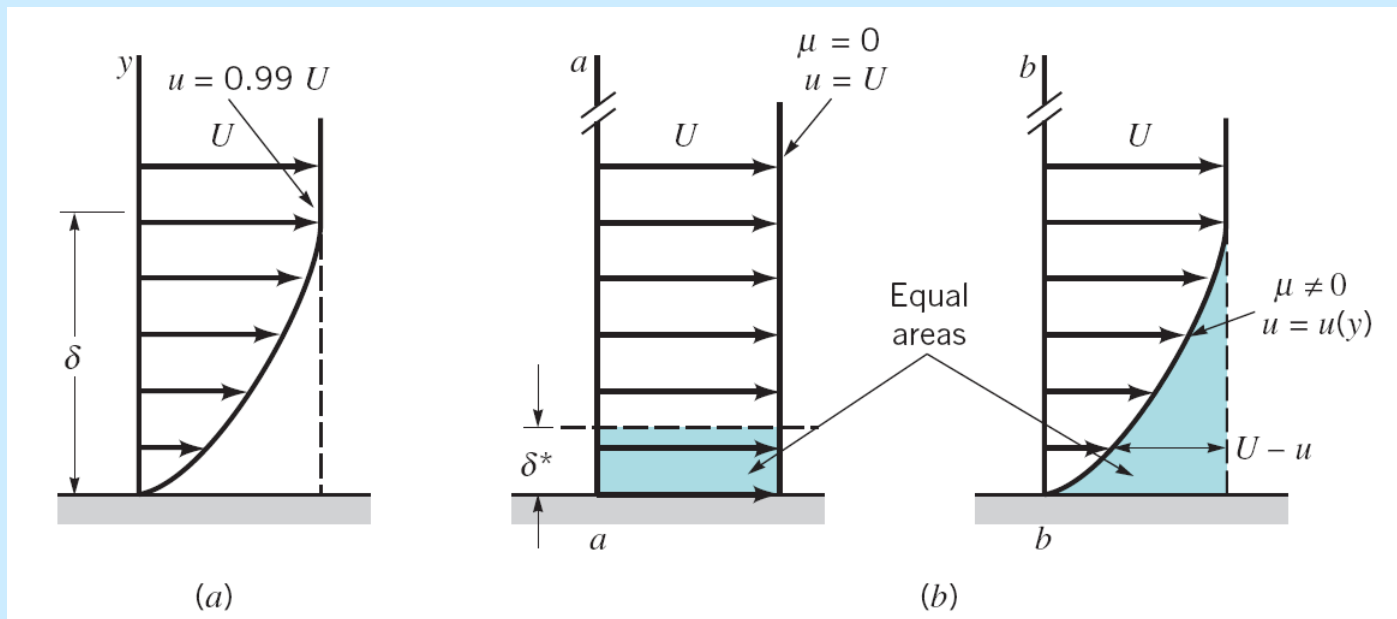
Ex 9.3 Determine the velocity $U=U(x)$ of the air within the duct but outside of the boundary layer with $\delta^* = 0.004(x)^{1/2}$

Boundary layer momentum thickness

- Boundary layer momentum thickness θ

$$\int \rho u\left(U-u\right)dA = \rho b\int_0^\infty u(U-u)dy = \rho bU^2\theta$$

$$\theta = \int_0^\infty \frac{u}{U}\left(1-\frac{u}{U}\right)dy$$

(9.4)

9.2.2 Prandtl/Blasius Boundary Layer Solution

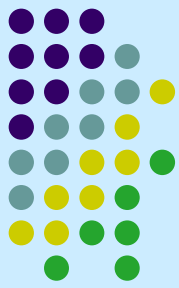- The Navier-Stokes equations are too complicated that no analytical solution is available. However, for large Re, *simplified* **boundary layer equations** can be derived.

- 2-D Navier-Stokes Equations

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \qquad (9.5)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \qquad (9.6)$$

Elliptic equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Boundary layer assumption

- The boundary layer assumption is based on the fact that the boundary layer is thin.

$$\delta \ll x \quad \text{so that} \quad v \ll u \quad \text{and} \quad \frac{\partial}{\partial x} \ll \frac{\partial}{\partial y}$$

Thus the equations become,

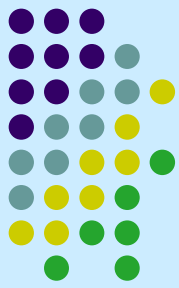$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad\qquad (9.8)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} \qquad\qquad (9.9)$$

parabolic equation

Detailed derivation is given in Cengel & Cimbala, Fluid Mechanics, 2006, pp. 516-519.

- Physically, the flow is parallel to the plate and any fluid is convected downstream much more quickly than it is diffused across the streamlines.

- For boundary layer flow over a flat plate the pressure is constant. The flow represents a balance between viscous and inertial effects, with pressure playing no role.
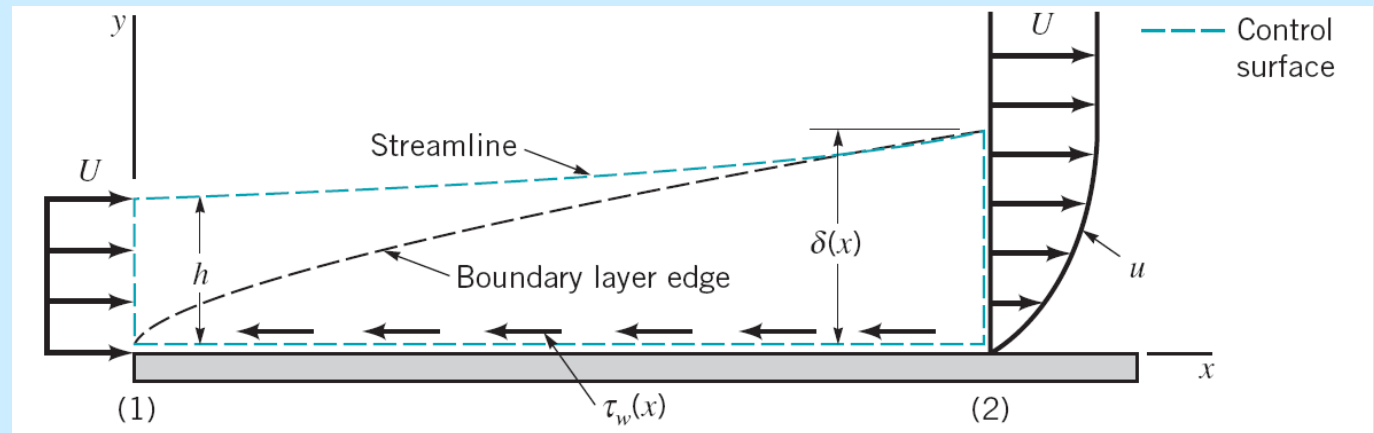
Boundary conditions

BCs: $u = v = 0$, $y = 0$

$u \to U$, $y \to \infty$

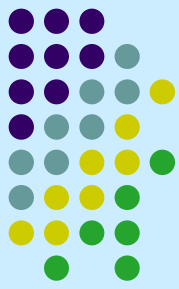It can be argued that the velocity profile should be similar $\to$ $\dfrac{u}{U} = g\left(\dfrac{y}{\delta}\right)$

In addition, order of magnitude analysis leads to:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}, \quad \frac{uu}{x} \sim \frac{\nu u}{\delta^2} ,$$

$$\delta^2 \sim \frac{\nu x}{U} , \quad \delta \sim \left(\frac{\nu x}{U}\right)^{1/2}$$

Similarity variables

Define the dimensionless similarity variable

$$\eta = \left(\frac{U}{\nu x}\right)^{\!1/2} y \sim \frac{y}{\delta},$$

and stream function

$$\psi = (\nu x U)^{1/2} f(\eta) \quad \text{where } f(\eta) \text{ is an unknown function}$$

Thus

$$\left\{ \begin{array}{l} u = \dfrac{\partial \psi}{\partial y} = (\nu x U)^{1/2} f'(\eta)\left(\dfrac{U}{\nu x}\right)^{\!1/2} = U f'(\eta) \\[4mm] v = -\dfrac{\partial \psi}{\partial x} = \left(\dfrac{\nu U}{4x}\right)^{\!1/2} (\eta f' - f) \quad \text{where } (\;)' = d\!\!\big/\!d\eta \end{array} \right.$$
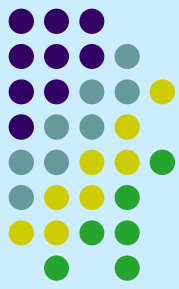
Then the parabolic equation (9.9) becomes

$$2f''' + ff'' = 0 \qquad\qquad\qquad\qquad (9.14\text{a})$$

with the boundary conditions

$$f(0) = f'(0) = 0 \ \text{ at } \eta = 0, \ f'(\infty) \to 1 \ \text{ as } \eta \to \infty \qquad (9.14\text{b})$$

Blasius solutions

$$\frac{u}{U} = f'(\eta)$$

from the numerical solution $\frac{u}{U} \approx 0.99$ $\eta = 5.0$ (Table 9.1)

$$\eta = 5 = \left(\frac{U}{\nu x}\right)^{\frac{1}{2}} \delta, \quad \delta = 5\sqrt{\frac{\nu x}{U}}$$

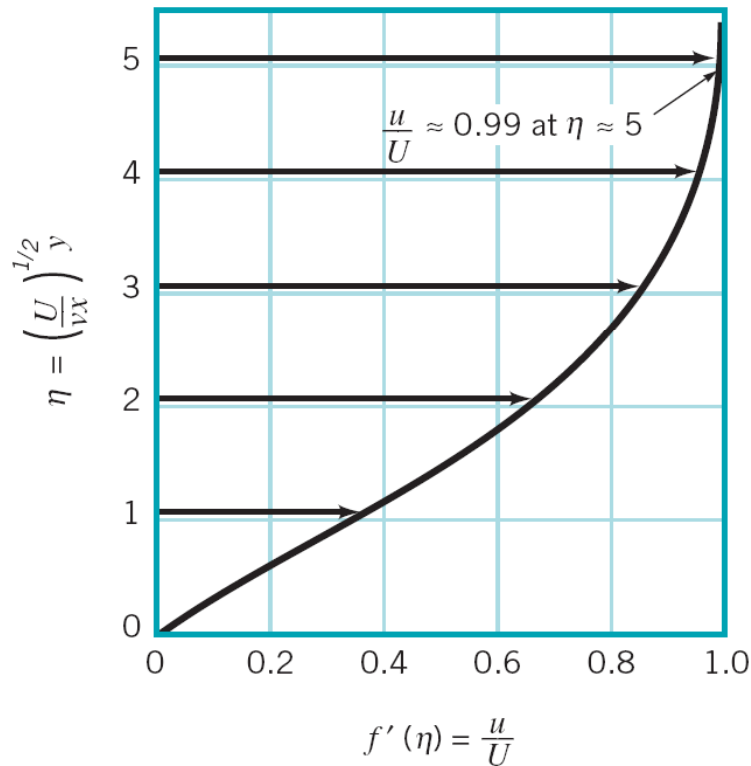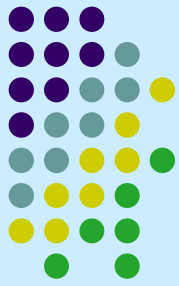or $\quad \dfrac{\delta}{x} = \dfrac{5}{\sqrt{Re_x}} \qquad \sqrt{Re_x} = \dfrac{Ux}{\nu}$

TABLE 9.1

Laminar Flow along a Flat Plate (the Blasius Solution)

$\eta = y(U/\nu x)^{1/2}$	$f'(\eta) = u/U$	η	$f'(\eta)$
0	0	3.6	0.9233
0.4	0.1328	4.0	0.9555
0.8	0.2647	4.4	0.9759
1.2	0.3938	4.8	0.9878
1.6	0.5168	5.0	0.9916
2.0	0.6298	5.2	0.9943
2.4	0.7290	5.6	0.9975
2.8	0.8115	6.0	0.9990
3.2	0.8761	∞	1.0000

HW: Derive (9.14&9.18) and solve using Matlab to get Table 9.1
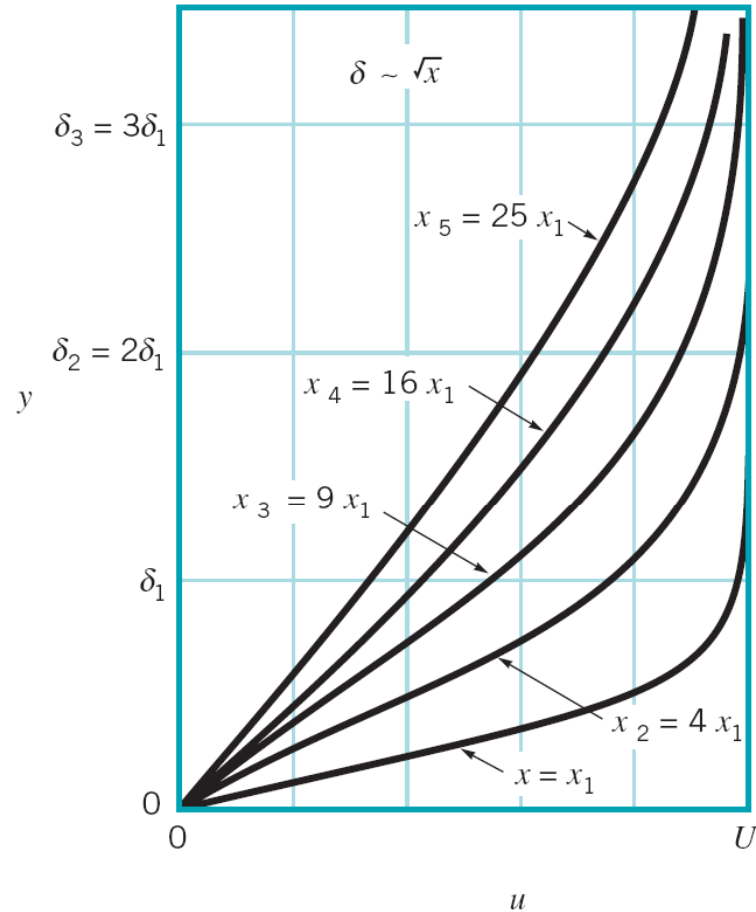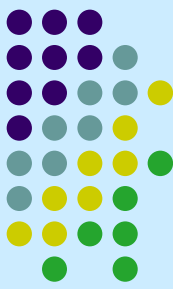
Blasius solution

Figure 9.10
Blasius boundary layer profile: (*a*) boundary layer profile in dimensionless form using the similarity variable ŋ. (*b*) similar boundary layer profiles at different locations along the flat plate.

Blasius solution

displacement thickness: from (9.3) $\rightarrow$ $\dfrac{\delta^*}{x} = \dfrac{1.721}{\sqrt{\mathrm{Re}_x}}$
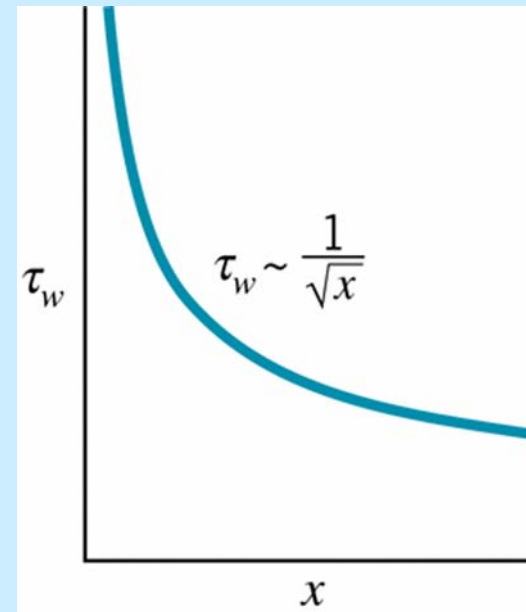
(9.16)

momentum thickness: from (9.4) $\rightarrow$ $\dfrac{\theta}{x} = \dfrac{0.664}{\sqrt{\mathrm{Re}_x}}$
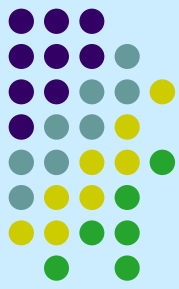
(9.17)

boundary layer Shear stress:

$$\tau_w = \mu \left. \dfrac{\partial u}{\partial y} \right|_{y=0} = 0.332 U^{3/2} \sqrt{\dfrac{\rho\mu}{x}}$$

(9.18)

$\tau_w \sim \dfrac{1}{\sqrt{x}}$

Note: For fully developed pipe flow shear stress, $\tau_w \propto U$ (why?)

9.2.3 Momentum-Integral Boundary Layer Equation for a Flat Plate

- One of the important aspects of boundary layer theory is the determination of the drag caused by shear forces on a body.
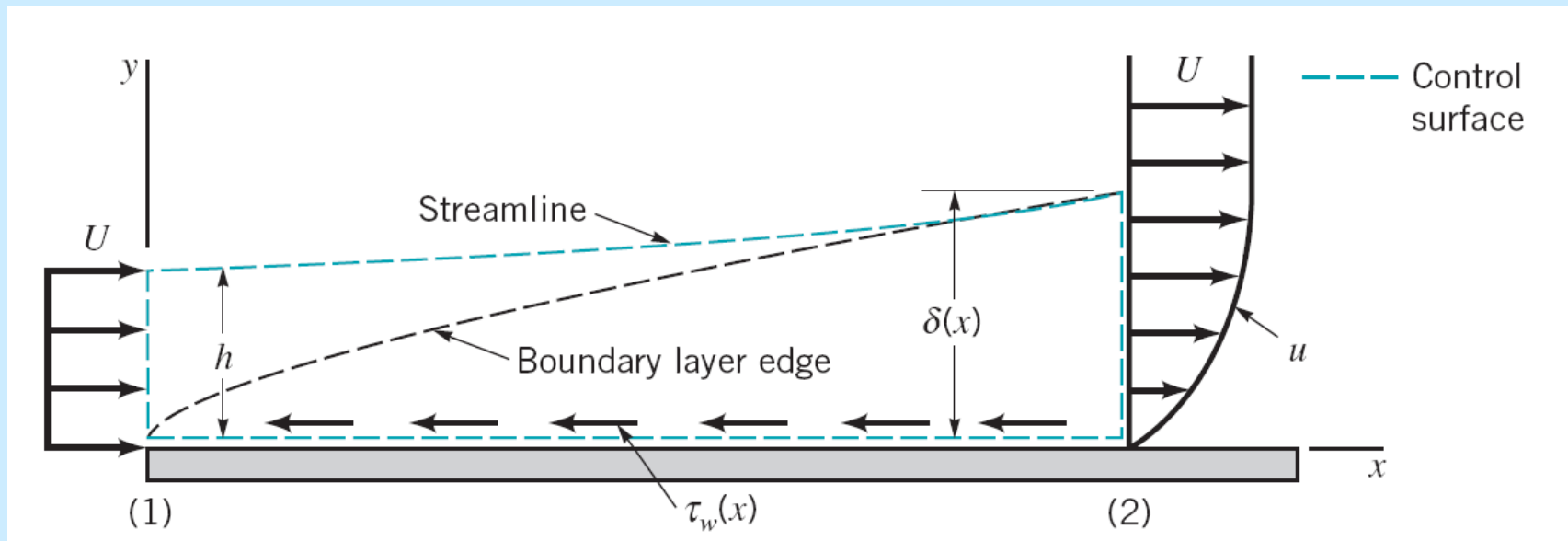
- Consider a uniform flow past a flat plate

Figure 9.11
Control volume used in the derivation of the momentum integral equation for boundary layer flow.

x-component of momentum equation:

$$\sum F_x = \rho \int u\vec{V}\cdot\vec{n}\,dA + \rho \int u\vec{V}\cdot\vec{n}\,dA$$
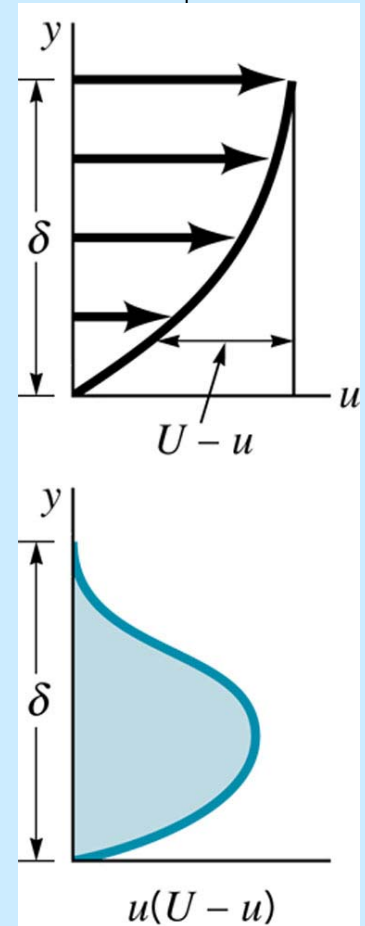$$\quad\;\; (1) \qquad\qquad (2)$$

$$\sum F_x = -D = -\int_{plate} \tau_w\,dA = -b\int_{plate} \tau_w\,dx \quad \text{or}$$

$$-D = \rho \int_{(1)} U(-U)\,dA + \rho \int_{(2)} u^2\,dA \;\rightarrow\; D = \rho U^2 bh - \rho b \int_0^{\delta} u^2\,dy$$

$$\text{Since} \quad Uh = \int_0^{\delta} u\,dy, \quad \rho U^2 bh = \rho b U \int_0^{\delta} u\,dy = \rho b \int_0^{\delta} Uu\,dy$$

$$D = \rho U^2 bh - \rho b \int_0^{\delta} u^2\,du = \rho b \int_0^{\delta} Uu\,dy - \rho b \int_0^{\delta} u^2\,dy = \rho b \int_0^{\delta} u(U-u)\,dy$$

$$D = \rho b U^2 \int_0^{\delta} \frac{u}{U}\left(1-\frac{u}{U}\right)dy = \rho b U^2 \theta, \quad \text{and} \quad \frac{dD}{dx} = \rho b U^2 \frac{d\theta}{dx}$$

The increase in drag per length of the plate occurs at the expense of an increase of the momentum boundary layer thickness, which represents a decrease in the momentum of the fluid.

$$\because dD = \tau_w b\,dx \rightarrow \frac{dD}{dx} = b\tau_w \quad \therefore \tau_w = \rho U^2 \frac{d\theta}{dx} \rightarrow \text{momentum integral equation by von Karman}$$

Momentum integral equation

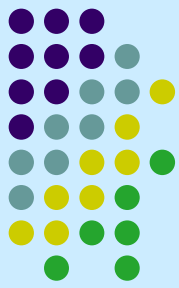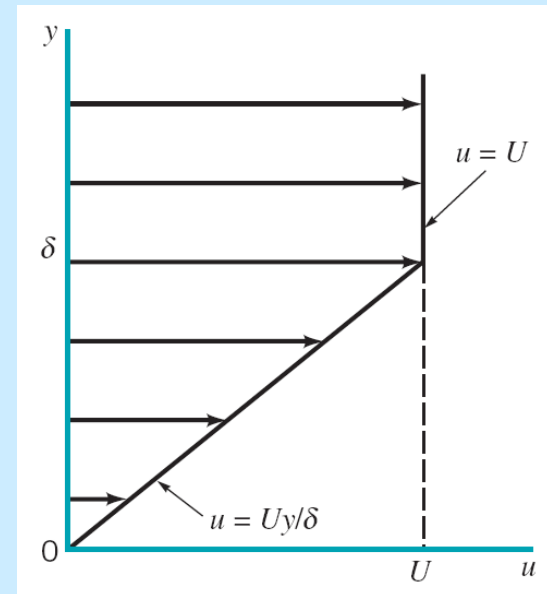$$\tau_w = \rho U^2 \frac{d\theta}{dx} \quad \rightarrow \quad \text{momentum integral equation}$$

- If we know the velocity distributions, we can obtain drag or shear stress.

- The accuracy of these results depends on how closely the shape of the assumed velocity profile approximates the actual profile.
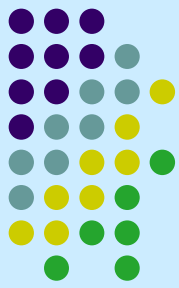
Example 9.4

$$u = \frac{Uy}{\delta} \quad 0 \le y \le \delta \quad u = U \quad y > \delta$$

$$\tau_w = \rho U^2 \frac{d\theta}{dx}$$

$$\tau_w = \mu \frac{U}{\delta}$$

$$\theta = \int_0^\infty \frac{u}{U}\left(1-\frac{u}{U}\right)dy = \int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right)dy$$

$$= \int_0^\delta \frac{y}{\delta}\left(1-\frac{y}{\delta}\right)dy = \delta\int_0^1\left(y-y^2\right)dy = \delta\left(\frac{y^2}{2}-\frac{y^3}{3}\right)\bigg|_0^1 = \frac{\delta}{6}$$

$$\frac{\mu U}{\delta} = \frac{\rho U^2}{6}\frac{d\delta}{dx} \quad , \quad \delta d\delta = \frac{6\mu}{\rho U}dx$$

$$\frac{\delta^2}{2} = \frac{6\mu}{\rho U}x \implies \delta^2 = \frac{12\mu}{\rho U}x$$

or $\delta = 3.46\sqrt{\dfrac{\mu x}{\rho U}}$ $\iff$ compare with Blasius solution $\dfrac{\delta}{x} = 5\sqrt{\dfrac{\mu}{\rho U x}}$

$$\frac{\delta}{x} = 3.46\sqrt{\frac{\mu}{\rho U x}}$$

$$\theta = \frac{\delta}{6} = 0.576\sqrt{\frac{\mu x}{\rho U}}$$

$$\tau_w = \rho U^2 \frac{d\theta}{dx} = 0.289 U^{3/2}\sqrt{\frac{\rho\mu}{x}} \iff \tau_w = 0.332 U^{3/2}\sqrt{\frac{\rho\mu}{x}}$$

TABLE 9.2
Flat Plate Momentum Integral Results for Various Assumed Laminar Flow Velocity Profiles

Profile Character	$\delta \mathrm{Re}_x^{1/2}/x$	$c_f \mathrm{Re}_x^{1/2}$	$C_{Df} \mathrm{Re}_\ell^{1/2}$
a. Blasius solution	5.00	0.664	1.328
b. Linear $u/U = y/\delta$	3.46	0.578	1.156
c. Parabolic $u/U = 2y/\delta - (y/\delta)^2$	5.48	0.730	1.460
d. Cubic $u/U = 3(y/\delta)/2 - (y/\delta)^3/2$	4.64	0.646	1.292
e. Sine wave $u/U = \sin[\pi(y/\delta)/2]$	4.79	0.655	1.310
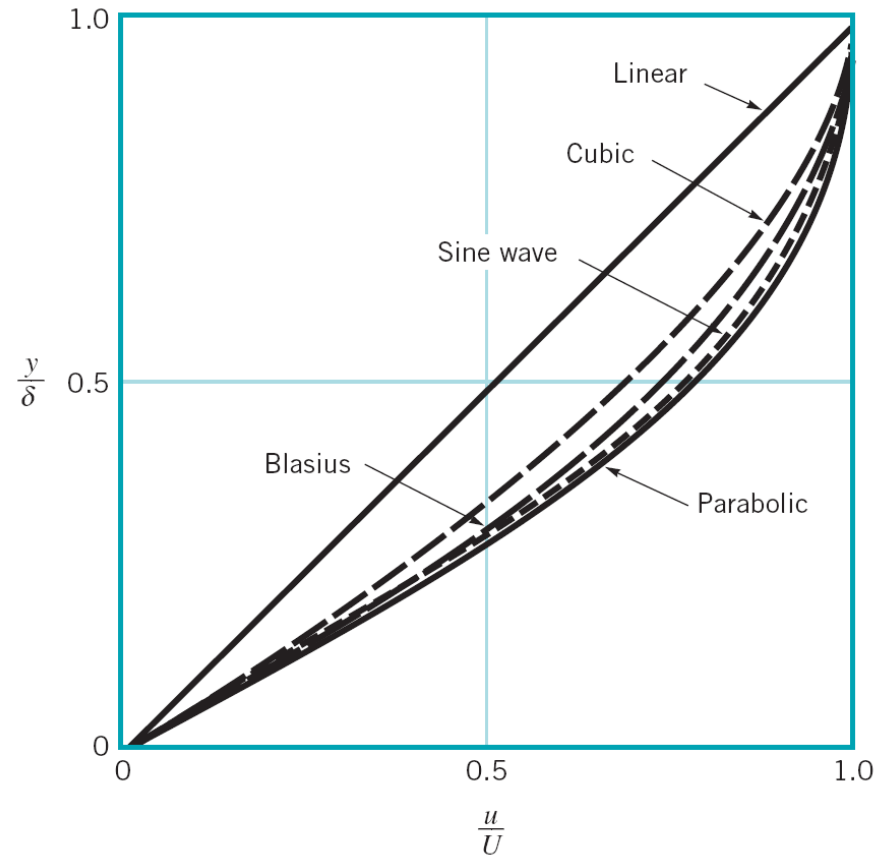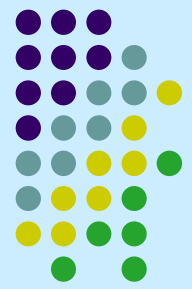
Figure 9.12
Typical approximate boundary layer profiles used in the momentum integral equation.

Momentum integral equation for general profile of *u/U*

Let $\dfrac{u}{U} = g(Y)$ for $0 \le Y \le 1$, $\dfrac{u}{U} = 1$ for $Y > 1$, where $Y = \dfrac{y}{\delta}$

B.C.: $g(0) = 0$, $g(1) = 1$

$$D = \rho b U^2 \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right)dy = \rho b U^2 \delta \int_0^1 g(Y)[1 - g(Y)]dY = \rho b U^2 \delta C_1$$

$$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = \frac{\mu U}{\delta}\left.\frac{dg}{dY}\right|_{Y=0} = \frac{\mu U}{\delta} C_2$$

we can obtain that

$$\boxed{\begin{array}{l} C_1 = \displaystyle\int_0^1 g(Y)[1 - g(Y)]dY \\[4mm] C_2 = \left.\dfrac{dg}{dY}\right|_{Y=0} \end{array}}$$
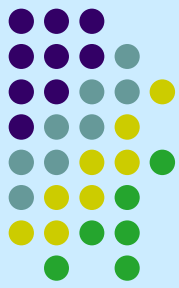
$$\frac{\delta}{x} = \frac{\sqrt{2C_2/C_1}}{\sqrt{\mathrm{Re}_x}} \quad \text{(Blasius solution: } \frac{\delta}{x} = \frac{5}{\sqrt{\mathrm{Re}_x}})$$

$$\tau_w = \frac{\mu U}{\delta} C_2 = \sqrt{\frac{C_1 C_2}{2}} U^{3/2} \sqrt{\frac{\rho \mu}{x}}$$

$$c_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \sqrt{2 C_1 C_2}\sqrt{\frac{\mu}{\rho U x}} = \frac{\sqrt{2 C_1 C_2}}{\sqrt{\mathrm{Re}_x}} \quad \text{(Blasius solution: } c_f = \frac{0.664}{\sqrt{\mathrm{Re}_x}})$$

$$C_{Df} = \frac{D_f}{\frac{1}{2}\rho U^2 b\ell} = \frac{b\displaystyle\int_0^\ell \tau_w \, dx}{\frac{1}{2}\rho U^2 b\ell} = \frac{\sqrt{8 C_1 C_2}}{\sqrt{\mathrm{Re}_\ell}}, \quad \text{where } \mathrm{Re}_l = \frac{\rho U \ell}{\mu} \quad \text{(Blasius solution: } C_{Df} = \frac{1.328}{\sqrt{\mathrm{Re}_\ell}})$$

9.2.4 Transition from Laminar to Turbulent Flow

- The analytical results are restricted to laminar boundary layer along a flat plate with zero pressure gradient.

- Transition to turbulent boundary layer occurs at

$$\mathrm{Re}_{x,\mathrm{cr}} = 2\times10^{5} \sim 3\times10^{6}$$

Example 9.5 Boundary layer transition

Figure 9.9

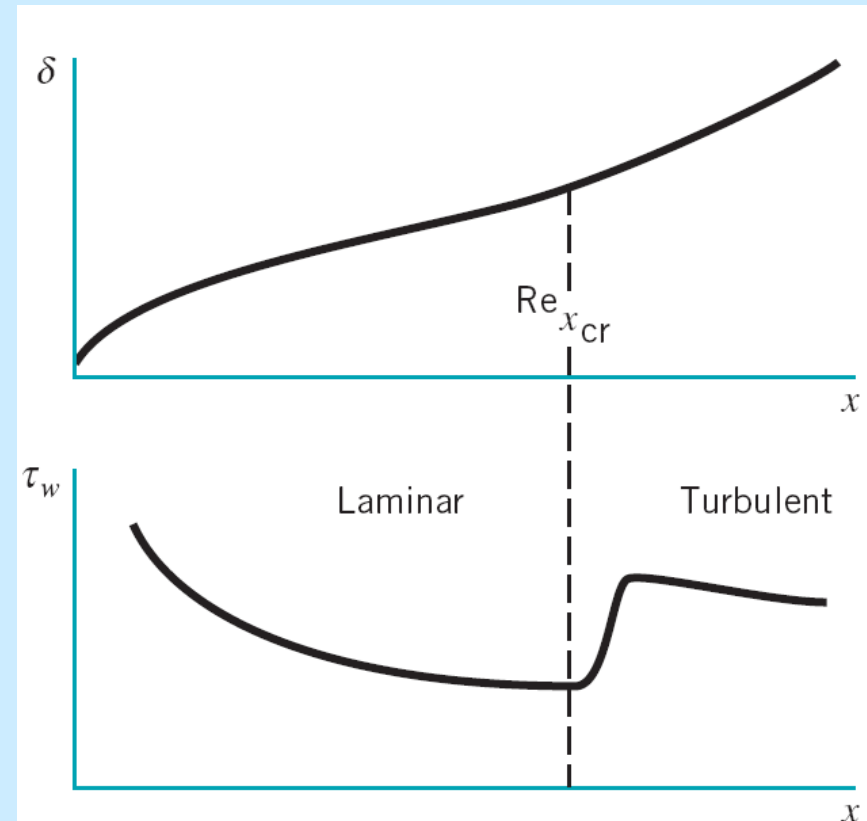**Typical characteristics of boundary layer thickness and wall shear stress for laminar and turbulent boundary layers.**

Figure 9.13 (p. 483)
Turbulent spots and the transition from laminar to turbulent boundary layer flow on a flat plate. Flow from left to right. (Photograph courtesy of B. Cantwell, Stanford University.

Figure 9.14 (p. 484)
Typical boundary layer profiles on a flat plate for laminar, transitional, and turbulent flow (Ref. 1).

V9.5 Transition on flat plate

9.2.5 Turbulent Boundary Layer Flow

- The structure of turbulent boundary layer flow is very complex, random, and irregular, with significant cross-stream mixing.

- Empirical power-law velocity profile is a reasonable approximation.

Plate
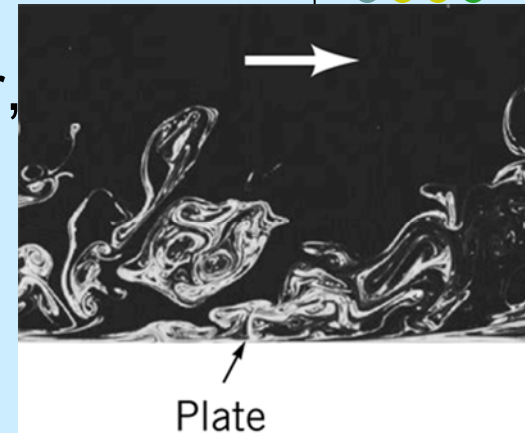
Example 9.6

To begin with the momentum integral equaion:

$$\tau_w = \rho U^2 \frac{d\theta}{dx}$$

Assume $\dfrac{u}{U} = \left(\dfrac{y}{\delta}\right)^{1/7} = Y^{1/7}$
$\begin{cases} Y = \dfrac{y}{\delta}, Y < 1 \\ u = U, Y > 1 \end{cases}$

and the experimentally determined formula:

$$\tau_w = 0.0225 \rho U^2 \left(\frac{\nu}{U\delta}\right)^{1/4} \qquad (1)$$
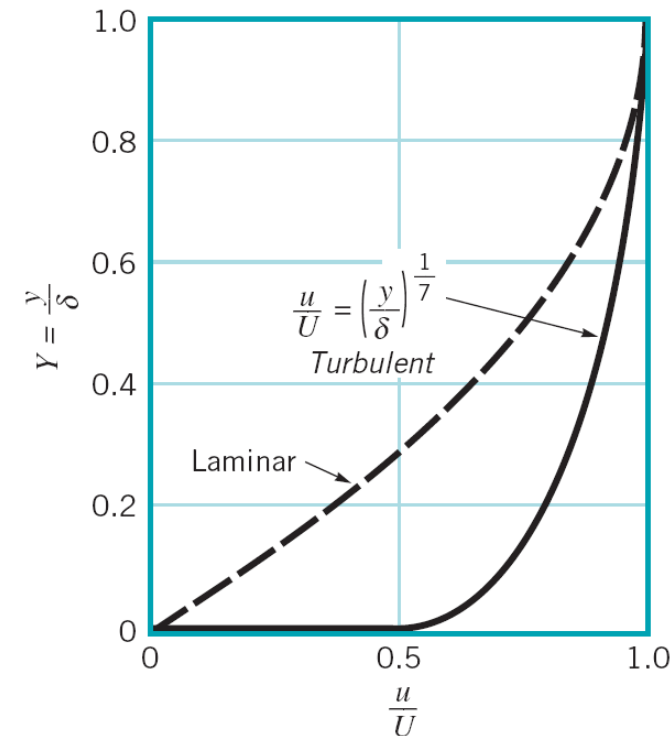
$$\tau_w = \rho U^2 \frac{d\theta}{dx}$$

$$\theta = \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right)dy = \delta\int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right)dY = \delta\int_0^1 Y^{1/7}\left(1 - Y^{1/7}\right)dY = \frac{7}{72}\delta \quad \text{(2)}$$

where δ is still unknown.

To determine δ, we combine Eq. 1 with Eq. 2 to get the ODE:

$$0.0225\rho U^2\left(\frac{\nu}{U\delta}\right)^{1/4} = \frac{7}{72}\rho U^2 \frac{d\delta}{dx} \quad \text{or} \quad \delta^{1/4}d\delta = 0.231\left(\frac{\nu}{U}\right)^{1/4}dx$$

with BC: $\delta = 0$ at $x = 0$, we obtain

$$\delta = 0.370\left(\frac{\nu}{U}\right)^{1/5}x^{4/5}, \quad \text{or} \quad \frac{\delta}{x} = \frac{0.370}{\text{Re}_x^{1/5}}$$

$$\delta^* = \int_0^\infty\left(1 - \frac{u}{U}\right)dy = \delta\int_0^1\left(1 - \frac{u}{U}\right)dY = \delta\int_0^1\left(1 - Y^{1/7}\right)dY = \frac{\delta}{8} = 0.0463\left(\frac{\nu}{U}\right)^{1/5}x^{4/5}$$

$$\theta = \frac{\delta}{72} = 0.036\left(\frac{\nu}{U}\right)^{1/5}x^{4/5}, \quad \theta < \delta^* < \delta$$

$$\tau_w = 0.0225\rho U^2 \left[\frac{\nu}{U\,0.37\left(\nu/U\right)^{1/5} x^{4/5}} \right]^{1/4} = \frac{0.0288\rho U^2}{\mathrm{Re}_x^{1/5}}$$

$$D_f = \int_0^\ell b\tau_w\,dx = b\,0.0288\rho U^2 \int_0^\ell \left(\frac{\nu}{Ux}\right)^{1/5} dx = 0.036\rho U^2 \frac{A}{\mathrm{Re}_\ell^{1/5}}, \qquad A = b\ell$$

$$C_{Df} = \frac{D_f}{\frac{1}{2}\rho U^2 A} = \frac{0.072}{\mathrm{Re}_\ell^{1/5}}$$
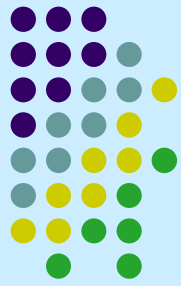
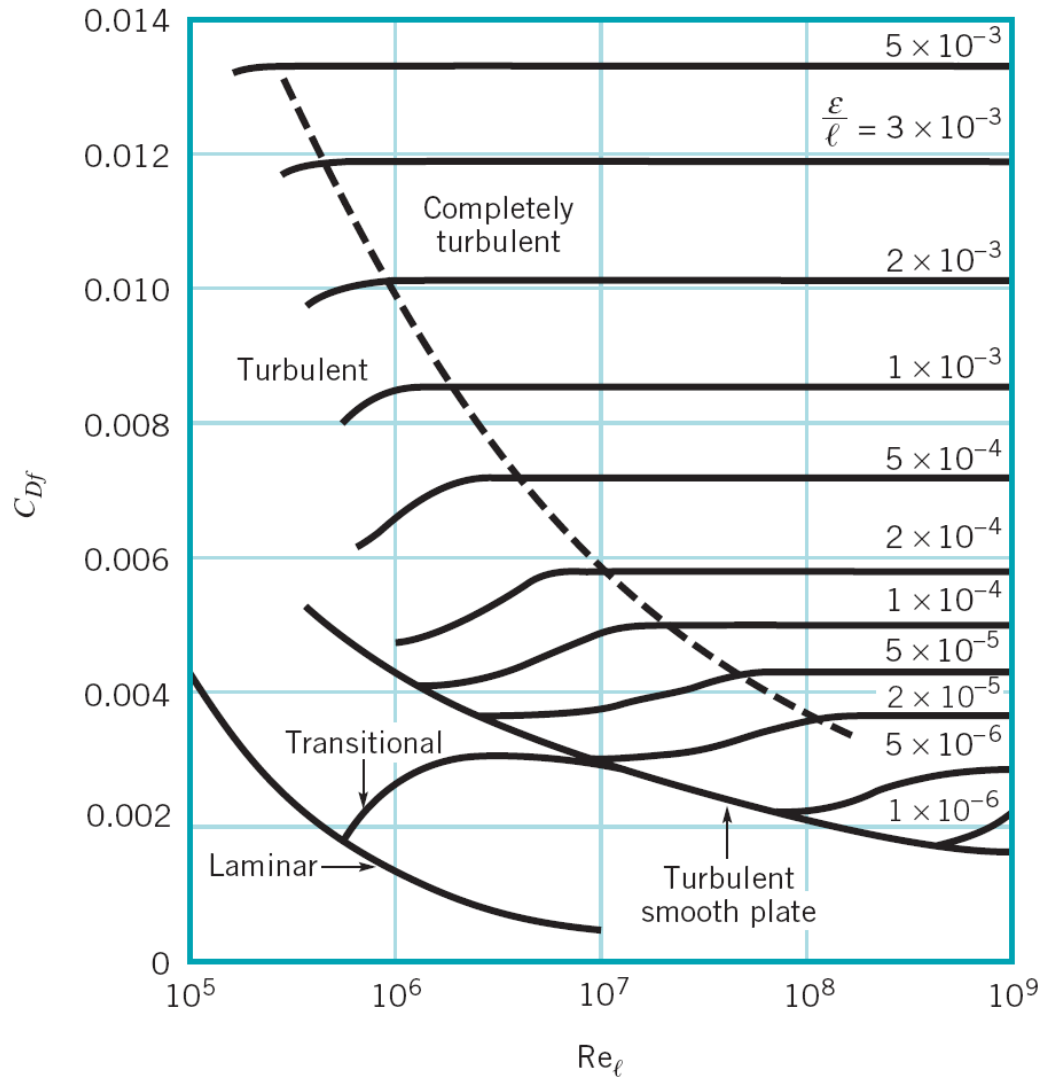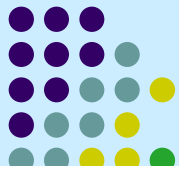**Note**: The above results are valid for smooth flat plates with

$$5\times 10^5 < \mathrm{Re}_x < 10^7$$

Turbulent B.L.: $\delta \square\, x^{4/5}, \tau_w \square\, x^{-1/5}$

Laminar B.L.: $\delta \square\, x^{1/2}, \tau_w \square\, x^{-1/2}$

Friction drag coefficient for a flat plate

Figure 9.15
Friction drag coefficient for a flat plate parallel to the upstream flow.
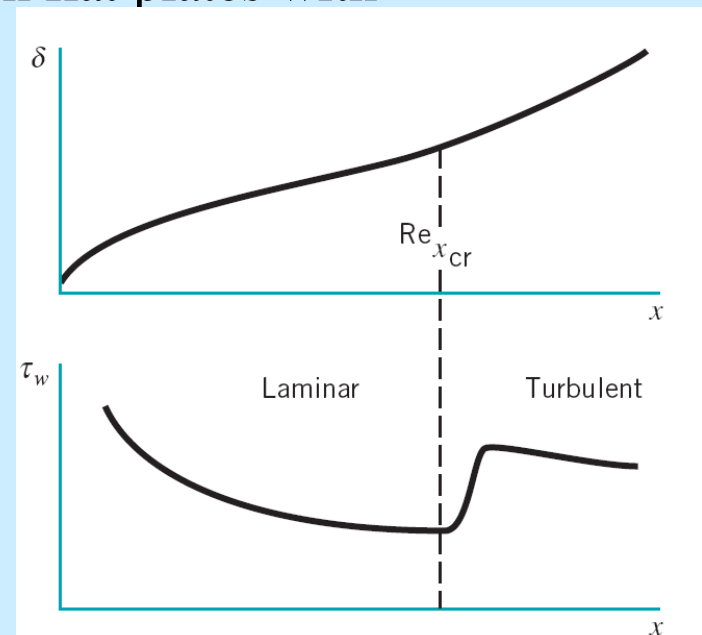
- Although Fig. 9.15 is similar with Moody diagram (pipe flow) of Fig. 8.23, the mechanisms are quite different.

For fully developed pipe flow, fluid inertia remains constant and the flow is balanced between pressure forces and viscous forces.

For flat plate boundary layer flow, pressure remains constant and the flow is balanced between inertia effects and viscous forces.

Empirical equations for C_{Df}
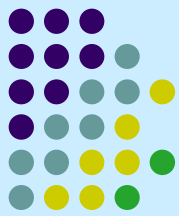
■ **TABLE 9.3**

Empirical Equations for the Flat Plate Drag Coefficient (Ref. 1)

Equation	Flow Conditions
$C_{Df} = 1.328/(\text{Re}_\ell)^{0.5}$	Laminar flow
$C_{Df} = 0.455/(\log \text{Re}_\ell)^{2.58} - 1700/\text{Re}_\ell$	Transitional with $\text{Re}_{xcr} = 5 \times 10^5$
$C_{Df} = 0.455/(\log \text{Re}_\ell)^{2.58}$	Turbulent, smooth plate
$C_{Df} = [1.89 - 1.62 \log(\varepsilon/\ell)]^{-2.5}$	Completely turbulent

Example 9.7

9.2.6 Effects of Pressure Gradient

Inviscid flow over a cylinder
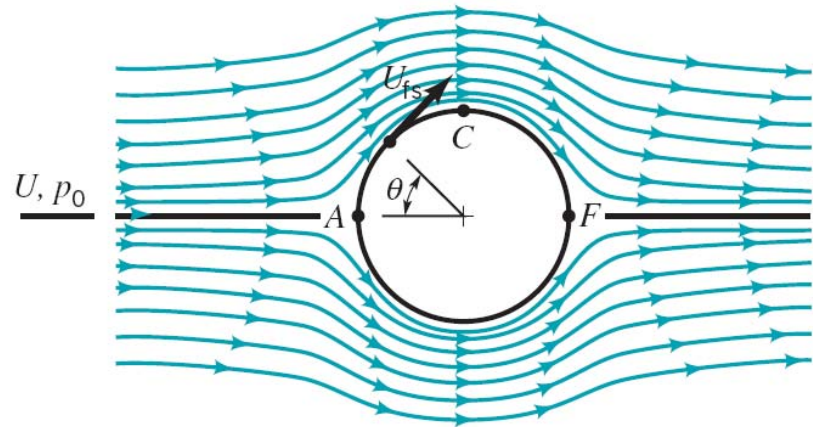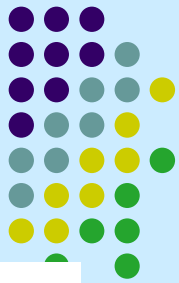
(a)
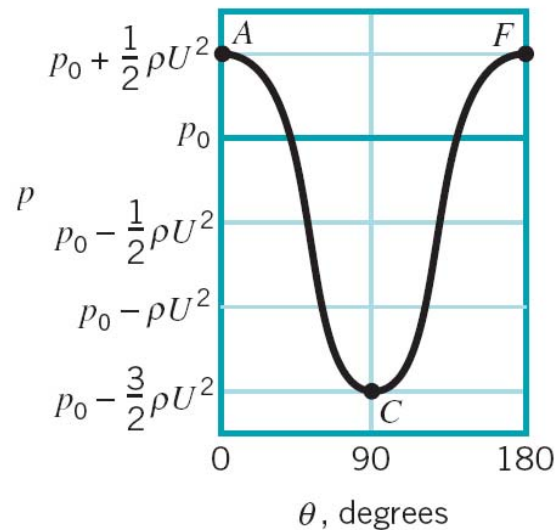
(b)
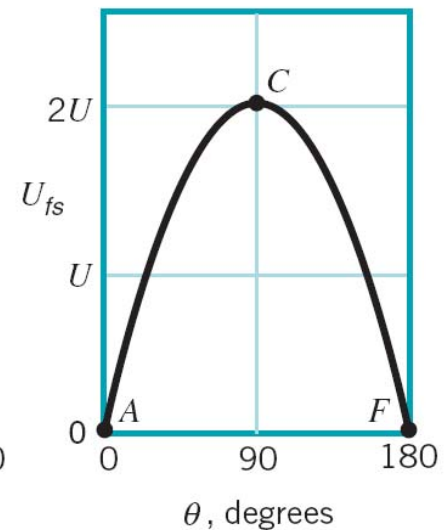
(c)

Viscous flow over a cylinder
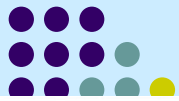
- Viscous flow
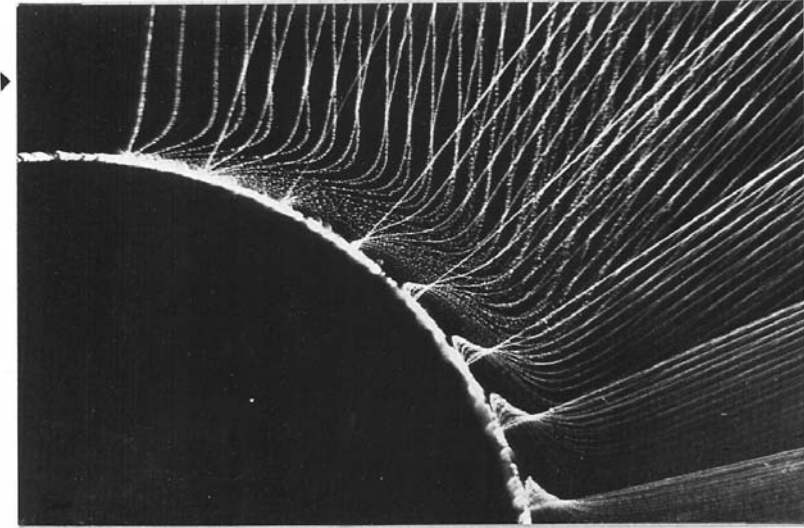
Fig. 22. Water, velocity of motion 2 cm/s, cylinder diameter 70 mm, photographed two seconds after the start of motion, $Re = 1.2 \times 10^3$, hydrogen bubble method.
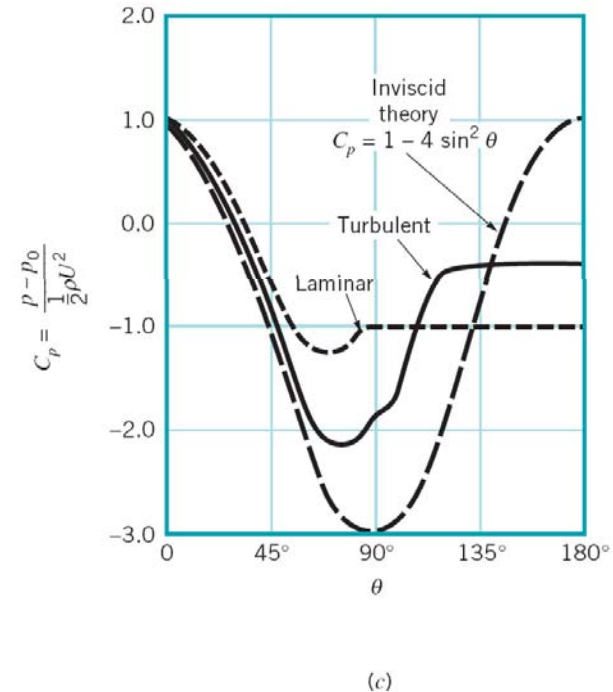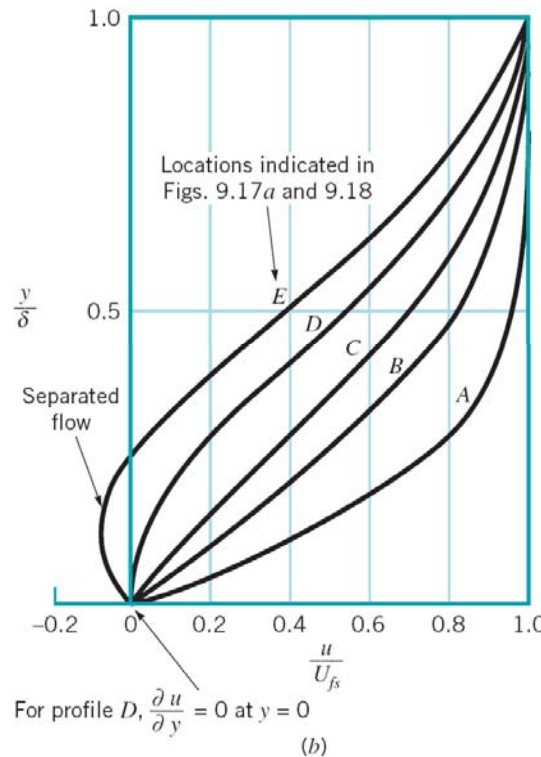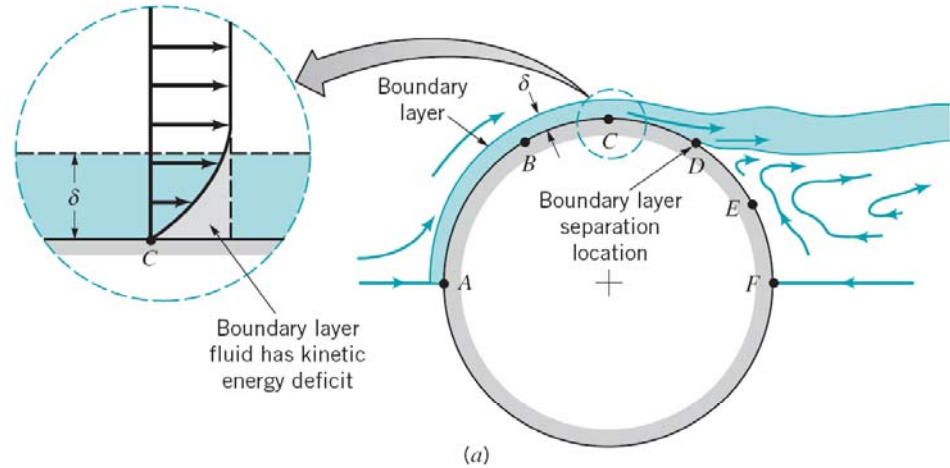
Boundary layer

δ

Boundary layer separation location

Boundary layer fluid has kinetic energy deficit

δ

C

B

C

D

E

A

F

(a)
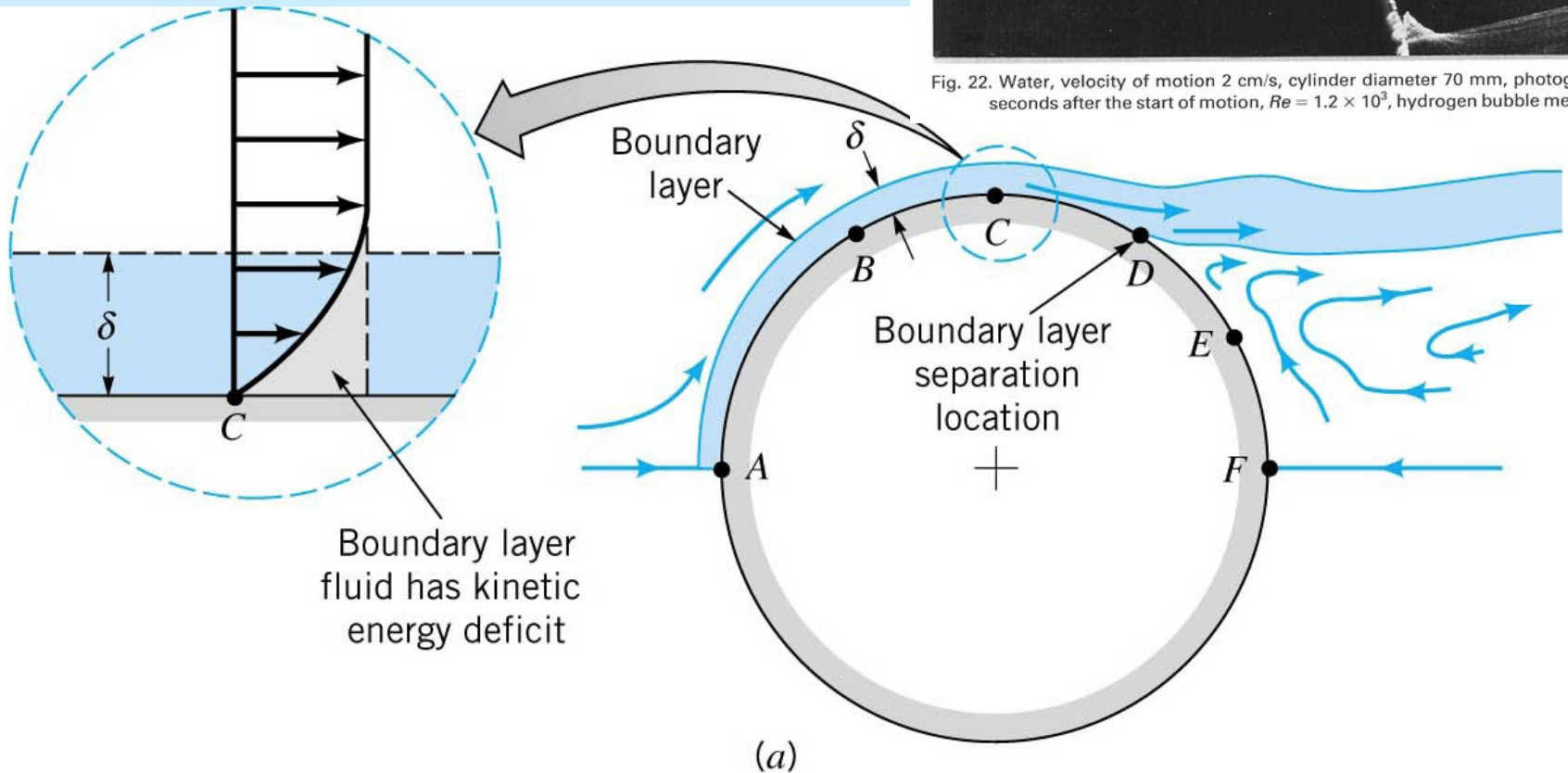
Viscous flow over cylinder: velocity-pressure
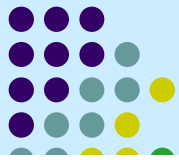
- Separated flow

- No matter how small the viscosity, provided it is not zero, there will be a boundary layer that separates from the surface, giving a drag that is, for the most part, independent of the value of μ.

9.2.7 Momentum-Integral Boundary Layer Equation with Nonzero Pressure Gradient
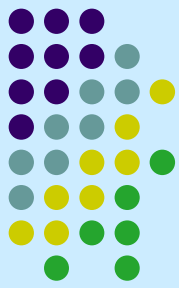
- Outside the boundary layer
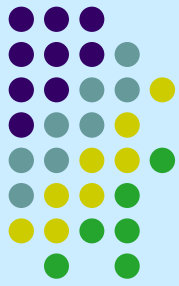
$$p + \frac{\rho U_{fs}^{2}}{2} = \text{constant}$$

$$\frac{dp}{dx} = -\rho U_{fs} \frac{dU_{fs}}{dx} \qquad (9.34)$$

- Momentum integral boundary layer equation

$$\tau_w = -\rho \frac{d}{dx}\left(U_{fs}^{2}\theta\right) + \rho \delta^* U_{fs} \frac{dU_{fs}}{dx} \Leftrightarrow (\text{for } U = C, \; \tau_w = \rho U^2 \frac{d\theta}{dx}) \quad (9.35)$$

- This equation represents a balance between viscous forces (τ_w), pressure forces($\frac{dp}{dx} = -\rho U_{fs} \frac{dU_{fs}}{dx}$), and the fluid momentum(θ).

- Eq. 9.35 can be used to provide information about the boundary layer thickness, wall shear stress, etc,

Derivation of equation (9.35)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \text{(a)}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{\partial U}{\partial x} + \frac{1}{\rho}\frac{\partial \tau}{\partial y} \quad \text{(b)}$$

$$\text{(b)} - (u - U)\text{(a)}$$

$$-\frac{1}{\rho}\frac{\partial \tau}{\partial y} = \frac{\partial}{\partial x}\left(uU - u^2\right) + (U - u)\frac{\partial U}{\partial x} + \frac{\partial}{\partial y}\left(vU - vu\right)$$

$$\int_0^\delta -\frac{1}{\rho}\frac{\partial \tau}{\partial y}\,dy = \frac{\partial}{\partial x}\int_0^\delta u(U-u)\,dy + \frac{\partial U}{\partial x}\int_0^\delta (U-u)\,dy + \int_0^\delta \frac{\partial}{\partial y}\left(vU - vu\right)\,dy$$

$v = 0$ at $y = 0, \delta$

$$\int_0^\delta -\frac{1}{\rho}\frac{\partial \tau}{\partial y}\,dy = \frac{\partial}{\partial x}\int_0^\delta u(U-u)\,dy + \frac{\partial U}{\partial x}\int_0^\delta (U-u)\,dy$$

$$\frac{\tau}{\rho} = \frac{\partial}{\partial x}\int_0^\delta u(U-u)\,dy + \frac{\partial U}{\partial x}\int_0^\delta (U-u)\,dy$$

$$= \frac{\partial}{\partial x}\left[U^2\int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right)dy + \frac{\partial U}{\partial x}U\int_0^\delta \left(1 - \frac{u}{U}\right)dy\right]$$

$$= \frac{\partial}{\partial x}\left(U^2\theta\right) + U\delta^*\frac{\partial U}{\partial x}$$
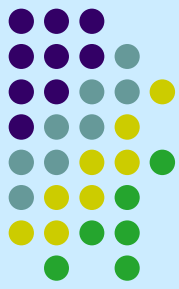
(9.35)

9.3 Drag

- Any object moving through a fluid will experience a drag *D* -- a net force in the direction of flow due to pressure (**pressure drag**) and shear stress (**friction drag**) on the surface of the object.

Drag coefficient:

$$C_D = \frac{D}{\frac{1}{2}\rho U^2 A} = \phi\left(\text{shape, Re, Ma, Fr, } \varepsilon/\ell\right)$$

These are determined experimentally, and very few can be obtained analytically.

9.3.1 Friction Drag

- Drag due to the shear stress τ_w, on the object.

- For large Re, the friction drag is generally smaller than pressure drag.

- However, for highly streamlined bodies or for low Reynolds number flow, most of the drag may be due to friction drag.

- Drag on a plate of width *b* and length *l*

$$D_f = \frac{1}{2} \rho U^2 b\ell C_{Df}$$

where C_{Df} is the friction drag coefficient.
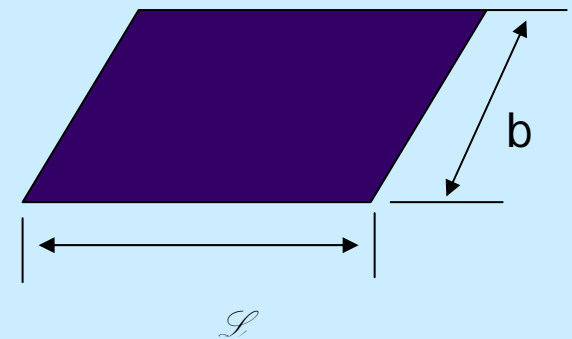
Friction Drag

For laminar flow, C_{Df} is independent of $\dfrac{\varepsilon}{D}$

For turbulent flow, C_{Df} is function of $\dfrac{\varepsilon}{D}$

C_{Df} along the surface of a curved body is quite difficult to obtain
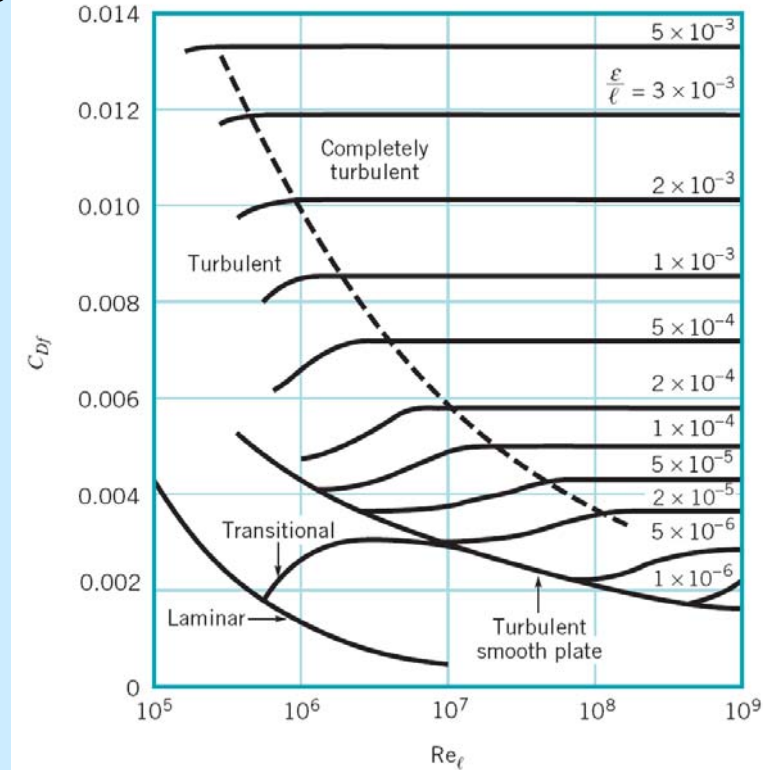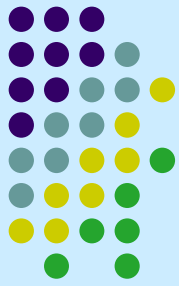
Example 9.8 Friction drag coeff. for a cylinder

9.3.2 Pressure Drag

- Drag due to pressure on the object.
- **Pressure drag** – also called **form drag** because of its strong dependency on the shape or form of the object.

$$D_p = \int p \cos \theta \, dA \quad C_p = \frac{p - p_0}{\frac{1}{2}\rho U^2} \; : \text{pressure coefficient}$$

$$C_{Dp} = \frac{D_p}{\frac{1}{2}\rho U^2 A} = \frac{\int p \cos \theta \, dA}{\frac{1}{2}\rho U^2 A} = \frac{\int C_p \cos \theta \, dA}{A} \quad p_0 : \text{reference pressure}$$

Pressure drag--Large Reynolds number

For large Reynolds number, $\left(\text{inertial effect} \gg \text{viscous effect}\right)$

$$p - p_0 \propto \frac{1}{2}\rho U^2 \ \left(\text{dynamic pressure}\right)$$

$$D_p = \int p\cos\theta dA \quad C_p = \frac{p - p_0}{\rho U^2 / 2} : \ \text{pressure coefficient}$$

$$C_{Dp} = \frac{D_p}{\frac{1}{2}\rho U^2 A} = \frac{\int p\cos\theta dA}{\frac{1}{2}\rho U^2 A} = \frac{\int C_p \cos\theta dA}{A}$$

Therefore, C_p is independent of Reynolds number,

C_{Dp} is also independent of Reynolds number $\left(\text{Re} \gg 1\right)$

Pressure drag--low Reynolds number
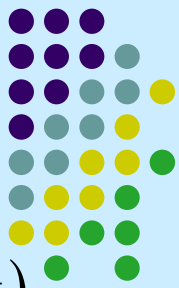
For very small Reynolds number $\left(\text{inertial effect} \square \text{ viscous effect}\right)$

$$\begin{cases} \square p \propto \mu \dfrac{U}{l} \ \left(\text{viscous stress}\right) \\ \tau_w \propto \mu \dfrac{U}{l} \end{cases}$$

$$C_{Dp} \ \square \ \dfrac{D_p}{\dfrac{1}{2}\rho U^2 A} \ \square \ \dfrac{\mu \dfrac{U}{l}}{\dfrac{1}{2}\rho U^2} \ \square \ \dfrac{\mu}{\rho U l} = \dfrac{1}{\text{Re}}$$

Comparisons:

For laminar pipe flow $f \ \square \ \dfrac{1}{\text{Re}}$

For large Reynolds number $f \ \square \ \text{constant} \ \left(\text{pipe flow}\right)$

Example 9.9 Pressure drag coeff. for a cylinder

Viscosity Dependence
Inviscid, Viscous flows over object
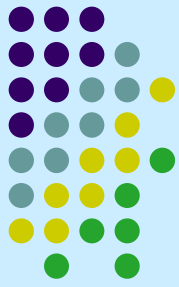
For $\mu = 0$, the pressure drag on any shaped object (symmetrical or not) in a steady flow would be zero.

For $\mu \neq 0$, the net pressure drag may be non-zero because of boundary layer separation.

(a)

(c)
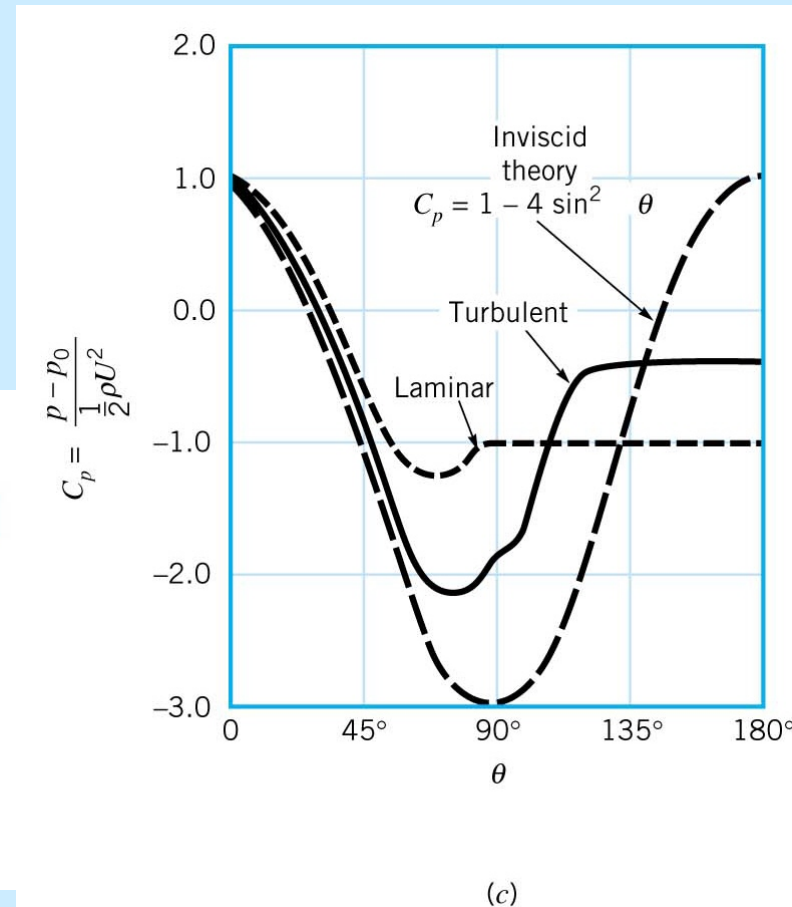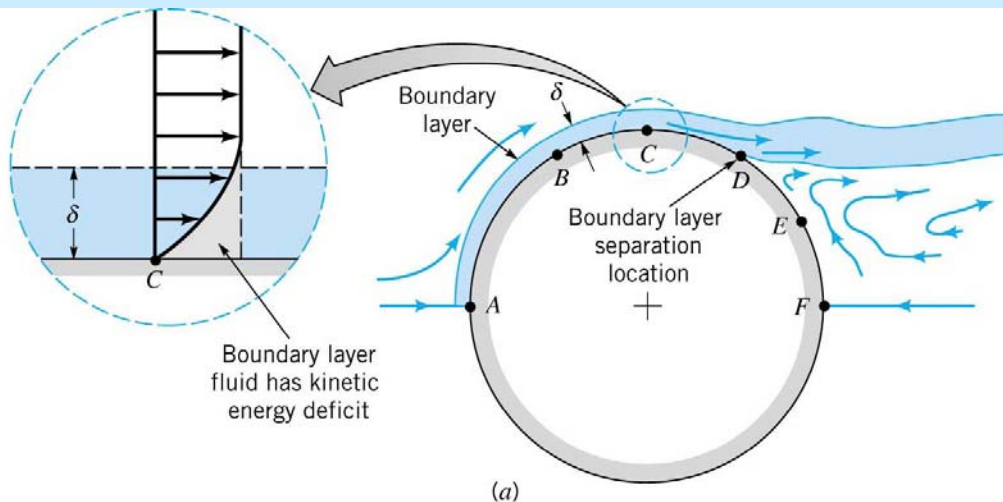
9.3.3 Drag Coefficient Data and Examples
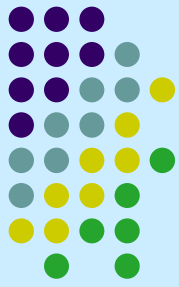
$$C_D = \phi\left(\text{shape, Re, Ma, Fr, } \frac{\varepsilon}{\ell}\right)$$

Shape Dependence
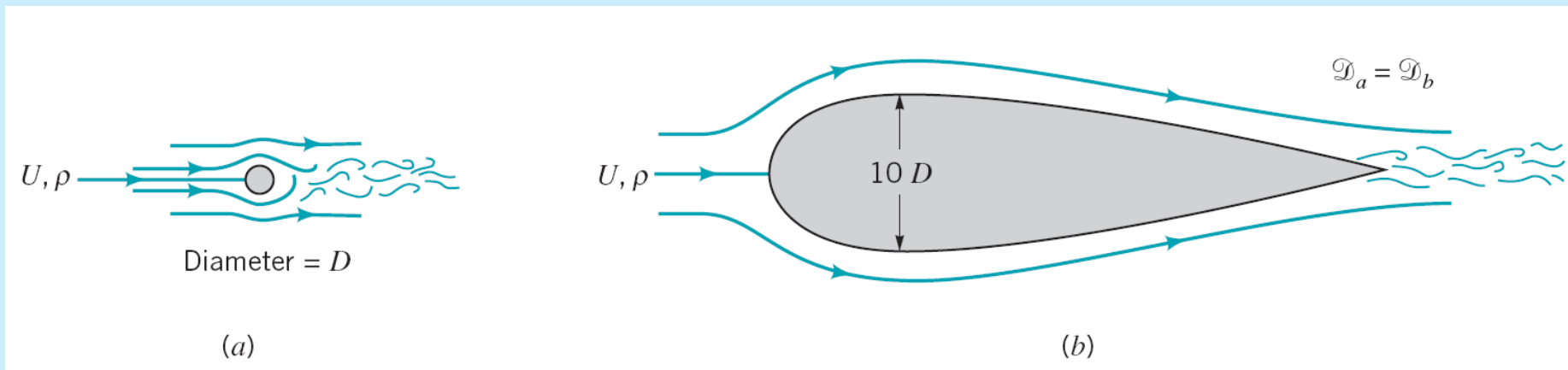
Figure 9.20
Two objects of considerably different size that have the same drag force: (a) circular cylinder $C_D = 1.2$; (b) streamlined strut $C_D = 0.12$.

Shape Dependence
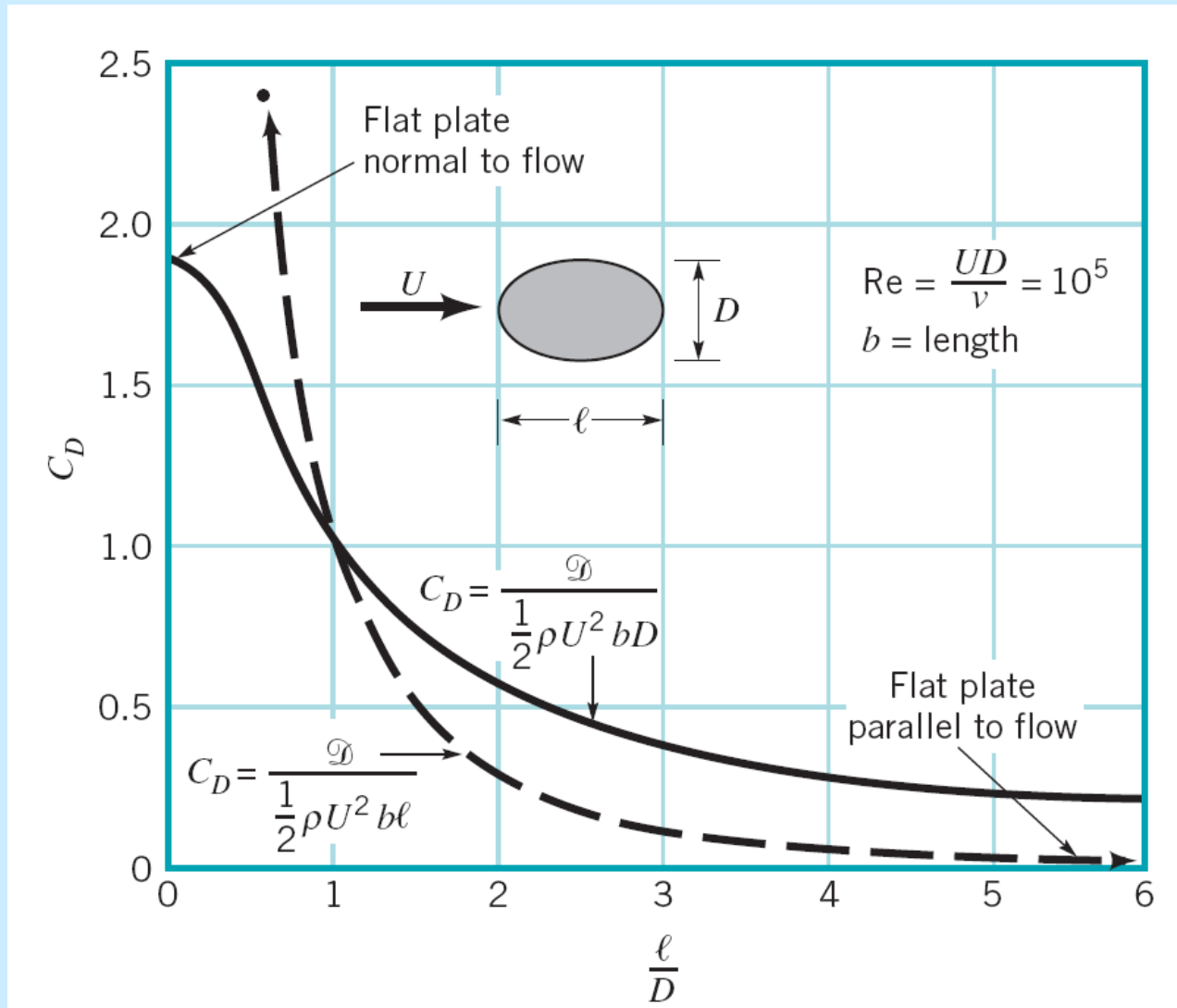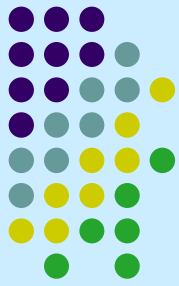
Figure 9.19
Drag coefficient for an ellipse with the characteristic area either the frontal area, _A_ = _b_D, or the planform area, _A_ = _bℓ_ (Ref. 5).

Reynolds Number Dependence

Low Reynolds number, Re<1

viscous force $\approx$ pressure force

$$D = f(U, l, \mu)$$

From dimensional analysis
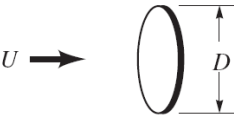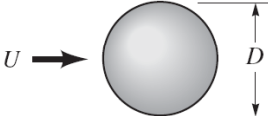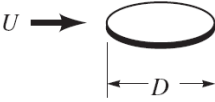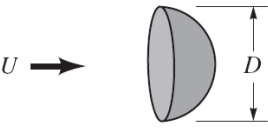
$$D = C \mu l U \quad [= C(\mu U / l) \cdot l^2]$$

$$C_D = \frac{D}{\frac{1}{2}\rho U^2 l^2} = \frac{2C\mu l U}{\rho U^2 l^2} = \frac{2C}{\text{Re}}, \quad \text{Re} = \frac{\rho U l}{\mu}$$
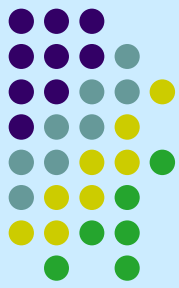
■ **T A B L E 9.4**

Low Reynolds Number Drag Coefficients (Ref. 7) (Re $= \rho UD/\mu$, $A = \pi D^2/4$)

Object	$C_D = \mathcal{D}/(\rho U^2 A/2)$ (for Re $\lesssim 1$)	Object	C_D
a. Circular disk normal to flow	20.4/Re	c. Sphere	24.0/Re
b. Circular disk parallel to flow	13.6/Re	d. Hemisphere	22.2/Re

Note: For Re<1, streamlining increases the drag due to an increase in the area on which shear force act.

Ex 9.10: particle in the water dragged up by upward motion   V9.7 Skydiving practice
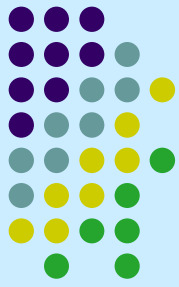
$D = 0.1 mm, \quad SG = 2.3, \quad$ to determine U

$W = D + F_B$

$W = \gamma_{sand} \forall = SG \gamma_{H_2O} \dfrac{\pi}{6} D^3, \qquad F_B = \gamma_{H_2O} \forall = \gamma_{H_2O} \dfrac{\pi}{6} D^3 ,$

$C_D = \dfrac{24}{\text{Re}} \ \left(For \ \text{Re} < 1 \right)$

$D = \dfrac{1}{2} \rho_{H_2O} U^2 \dfrac{\pi}{4} D^2 C_D = \dfrac{1}{2} \rho_{H_2O} U^2 \dfrac{\pi}{4} D^2 \left(\dfrac{24}{\rho_{H_2O} UD / \mu_{H_2O}} \right) = 3\pi \mu_{H_2O} UD$

$\therefore SG \gamma_{H_2O} \dfrac{\pi}{6} D^3 = 3\pi \mu_{H_2O} UD + \gamma_{H_2O} \dfrac{\pi}{6} D^3$

$\gamma = \rho g$

$U = \dfrac{\left(SG \rho_{H_2O} - \rho_{H_2O} \right) g D^2}{18 \mu} = 6.32 \times 10^{-3} \ {m}/{s}$

For $15.6°C, \ \rho_{H_2O} = 999 \ {kg}/{m^3} , \ \mu_{H_2O} = 1.12 \times 10^{-3} \ {N \cdot S}/{m^2}$

$\text{Re} = \dfrac{\rho UD}{\mu} = 0.564 \ \left(\text{Re} < 1 \right)$
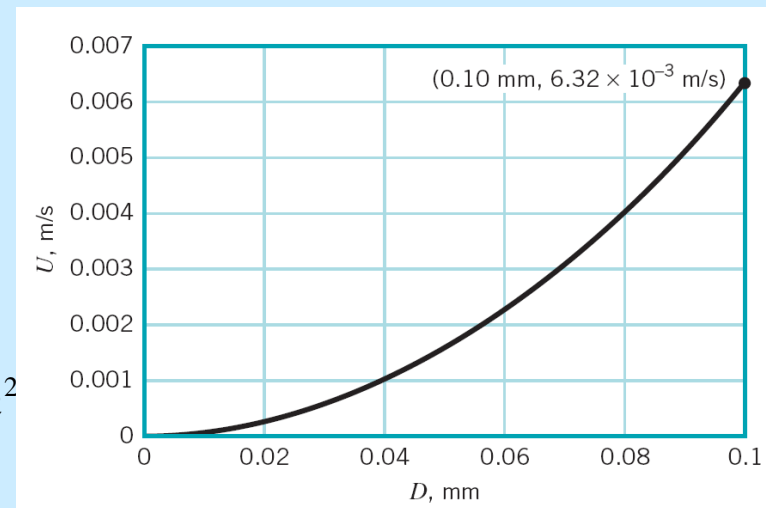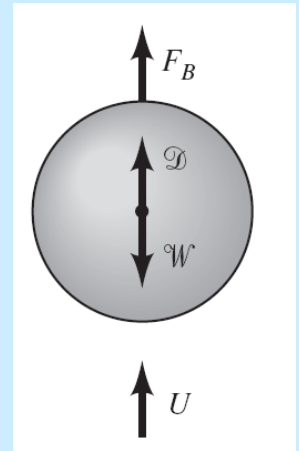
Moderate Reynolds Number
C_D of cylinder and sphere

$$\text{Re} < 10^3 \quad C_D \sim \text{Re}^{-1/2} \quad [\text{Re} \uparrow, C_D \downarrow (\text{not } D)]$$

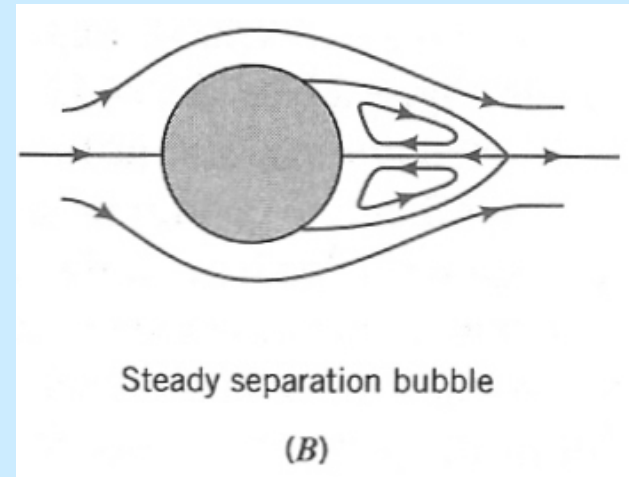$$10^3 < \text{Re} < 10^5 \quad C_D \sim \text{constant}$$

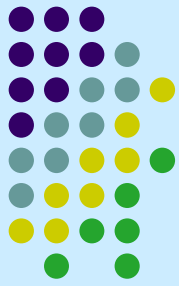$$\text{Re} = \frac{\rho U D}{\mu}$$
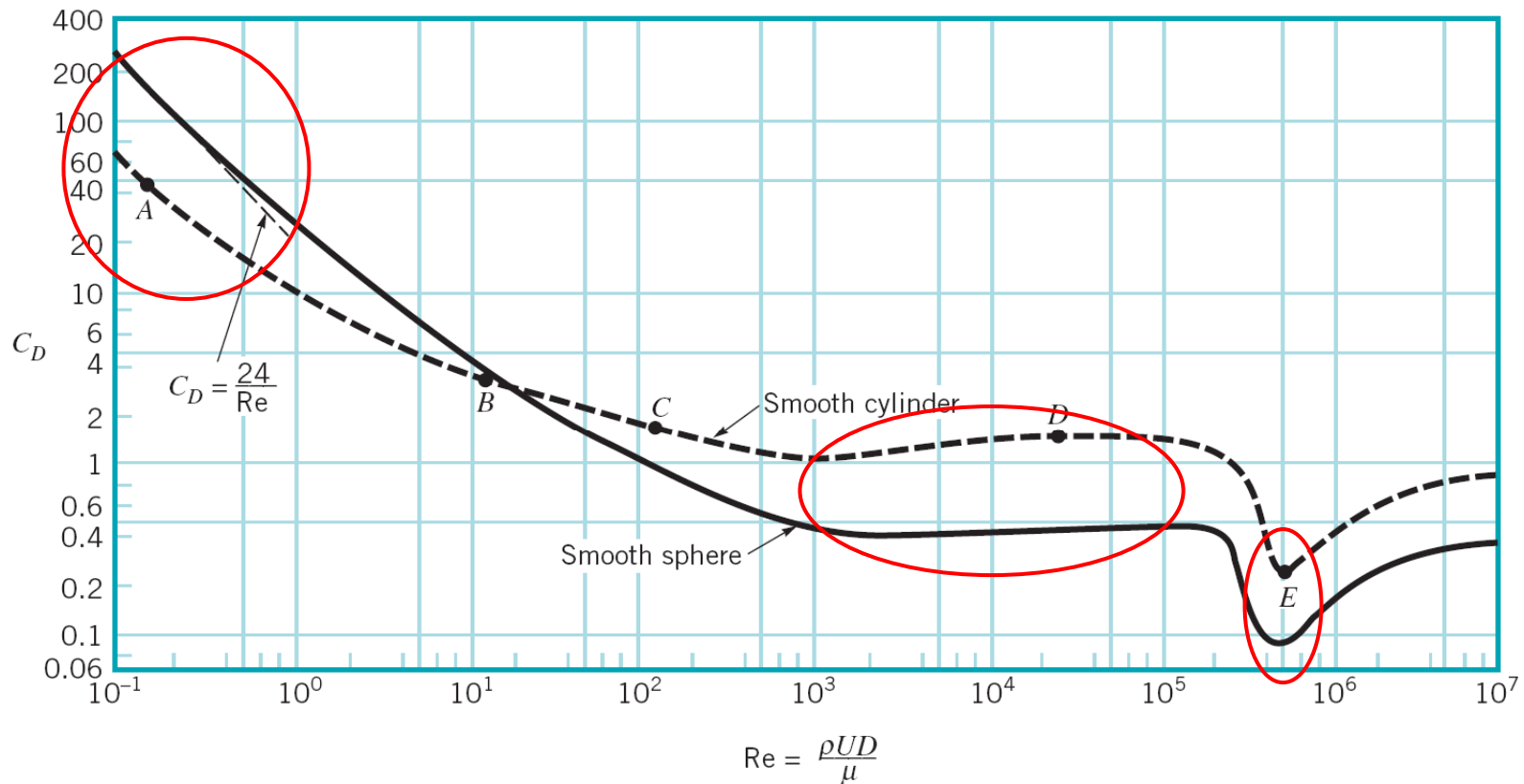
(a)

Flow over cylinder at various Reynolds number
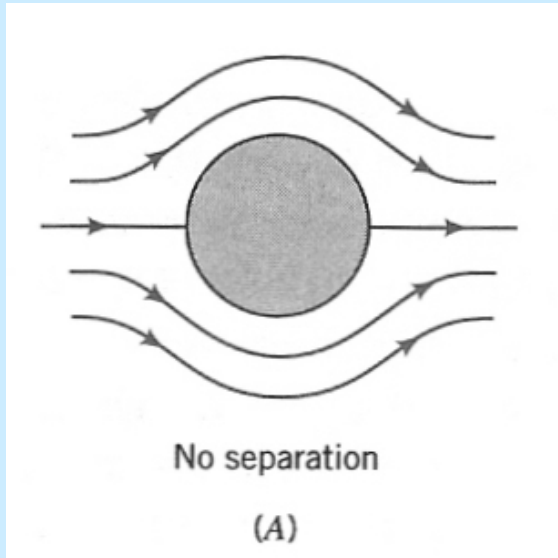
Re < 1 , no separation

Re ≈ 10, steady separation bubble

V9.8 Karman vortex street
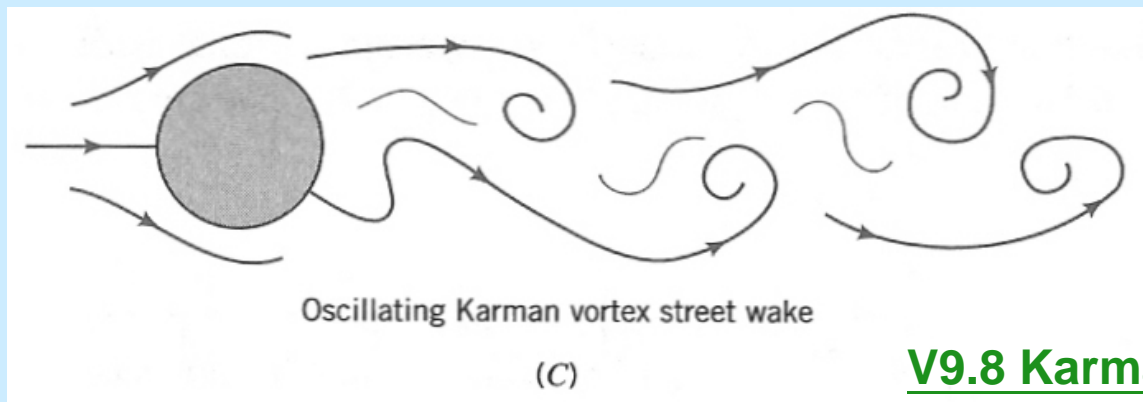
Re ≈ 100, vortex shedding starts at Re ≈ 47

Flow over cylinder at various Reynolds number

Laminar boundary layer, wide turbulent wake

(D)

$$Re \approx 5 \times 10^4$$

Turbulent boundary layer, narrow turbulent wake
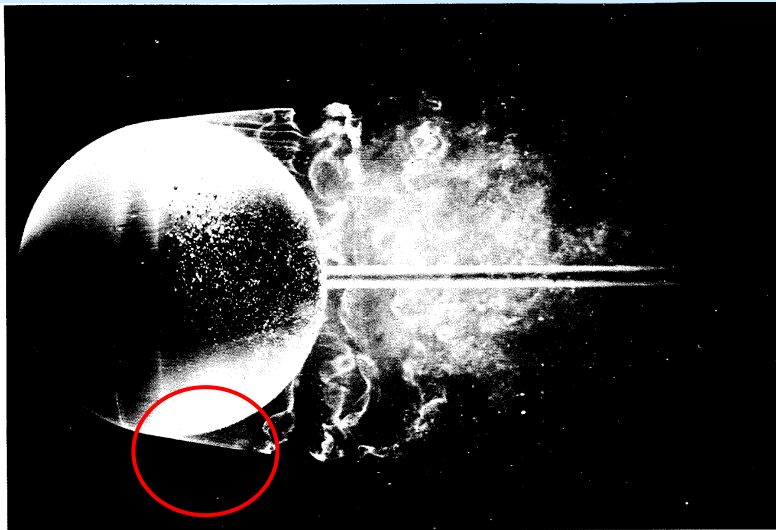
(E)
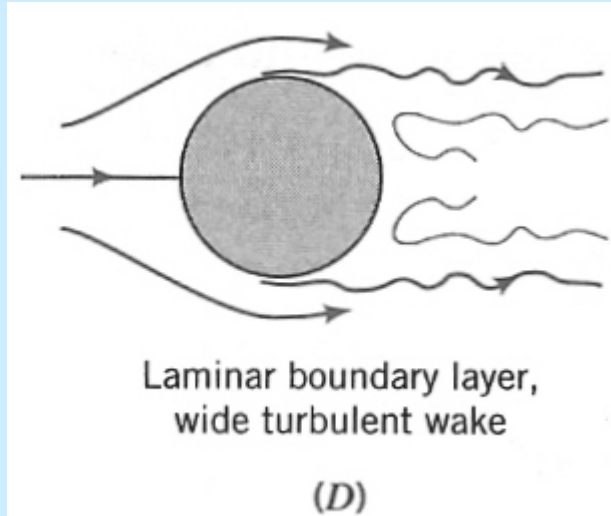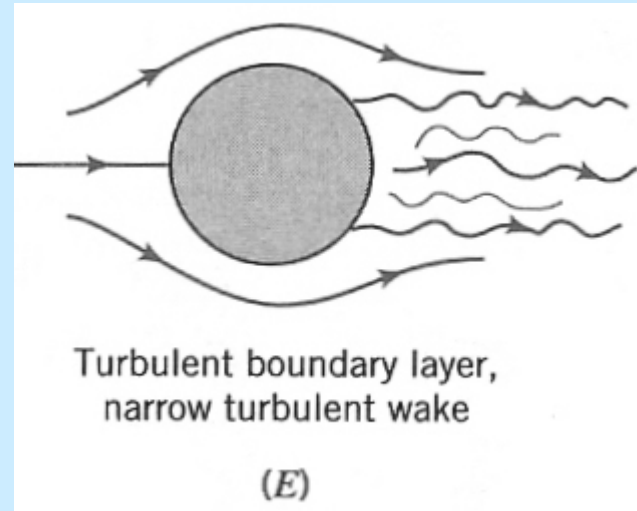
$$Re \approx 4 \times 10^5$$

from M. Van Dyke, An Album of Fluid Motion

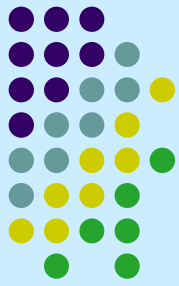**55. Instantaneous flow past a sphere at** $R=15,000$. Dye in water shows a laminar boundary layer separating ahead of the equator and remaining laminar for almost one radius. It then becomes unstable and quickly turns turbulent. *ONERA photograph, Werlé 1980*

57. Instantaneous flow past a sphere at $R=30,000$ **with a trip wire.** A classical experiment of Prandtl and Wieselsberger is repeated here, using air bubbles in water. A wire hoop ahead of the equator trips the boundary layer. It becomes turbulent, so that it separates farther rearward than if it were laminar (opposite page). The drag is thereby dramatically reduced, in a way that occurs naturally on a smooth sphere only at a Reynolds number ten times as great. *ONERA photograph, Werlé 1980*

Laminar and Turbulent Boundary Layer Separation
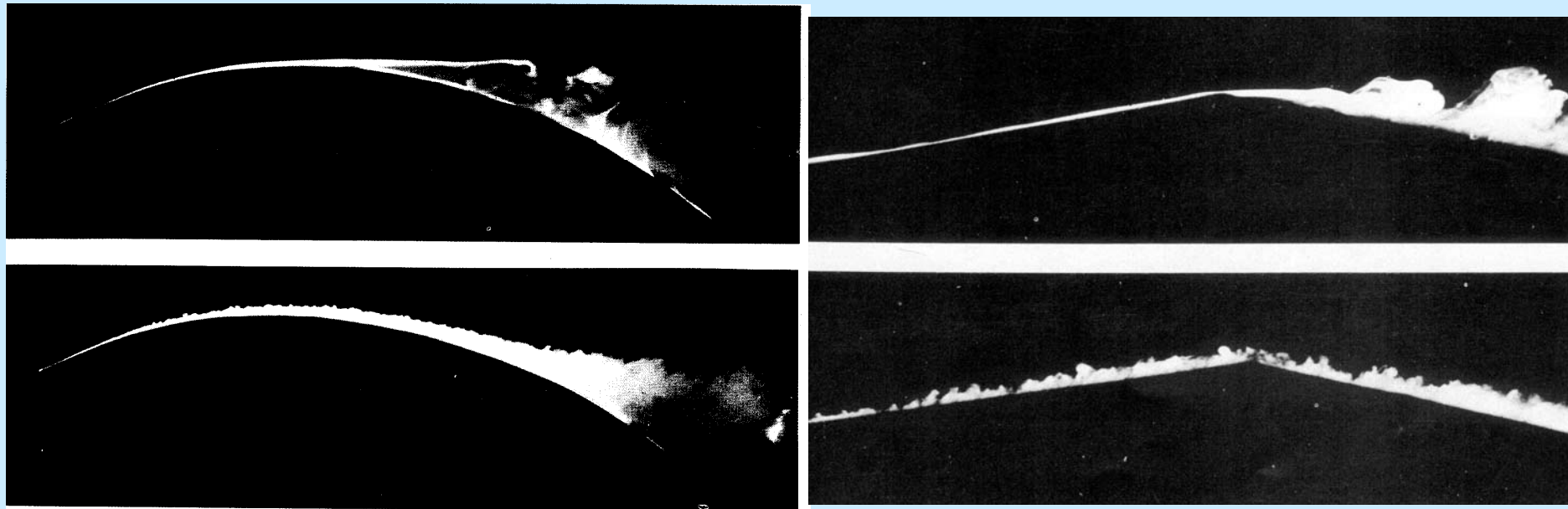
156. Comparison of laminar and turbulent boundary layers. The laminar boundary layer in the upper photograph separates from the crest of a convex surface (cf. figure 38), whereas the turbulent layer in the second photograph remains attached; similar behavior is shown below for a sharp corner. (Cf. figures 55-58 for a sphere.) Titanium tetrachloride is painted on the forepart of the model in a wind tunnel. *Head 1982*

from M. Van Dyke, An Album of Fluid Motion

Drag of streamlined and blunt bodies
Drag coefficients

The different relations of C_D with Re for objects with various streamlining are illustrated in Fig. 9.22.
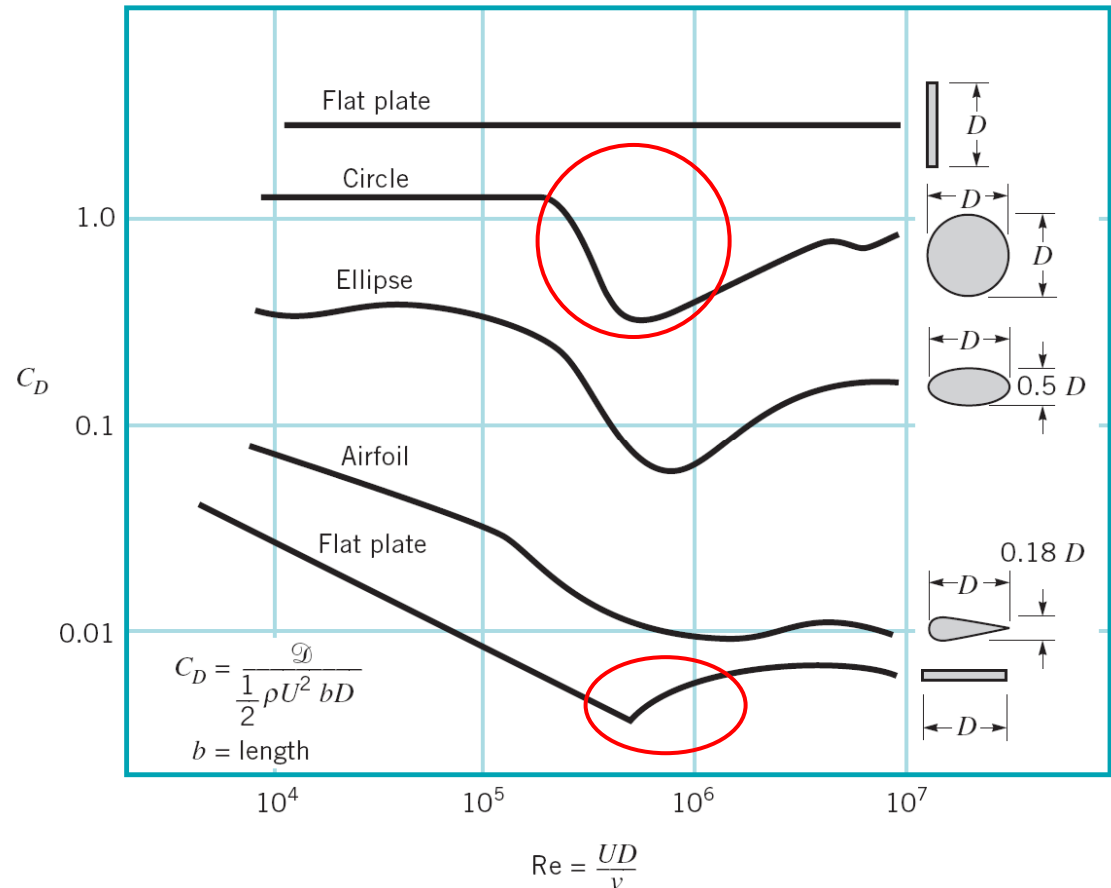
For streamlined bodies, C_D↑ when the boundary layer becomes turbulent.
For blunt objects, C_D↓ when the boundary layer becomes turbulent.

Note: In a portion of the range $10^5 < \mathrm{Re} < 10^6$, the actual drag (not just C_D) decreases with increasing speed.
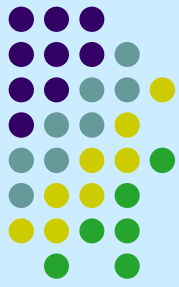
V9.9 Oscillating sign

V9.10 Flow past a flat plate

V9.11 Flow past an ellipse

$$C_D = \frac{\mathcal{D}}{\frac{1}{2}\rho U^2\, bD}$$

b = length

$$\mathrm{Re} = \frac{UD}{\nu}$$

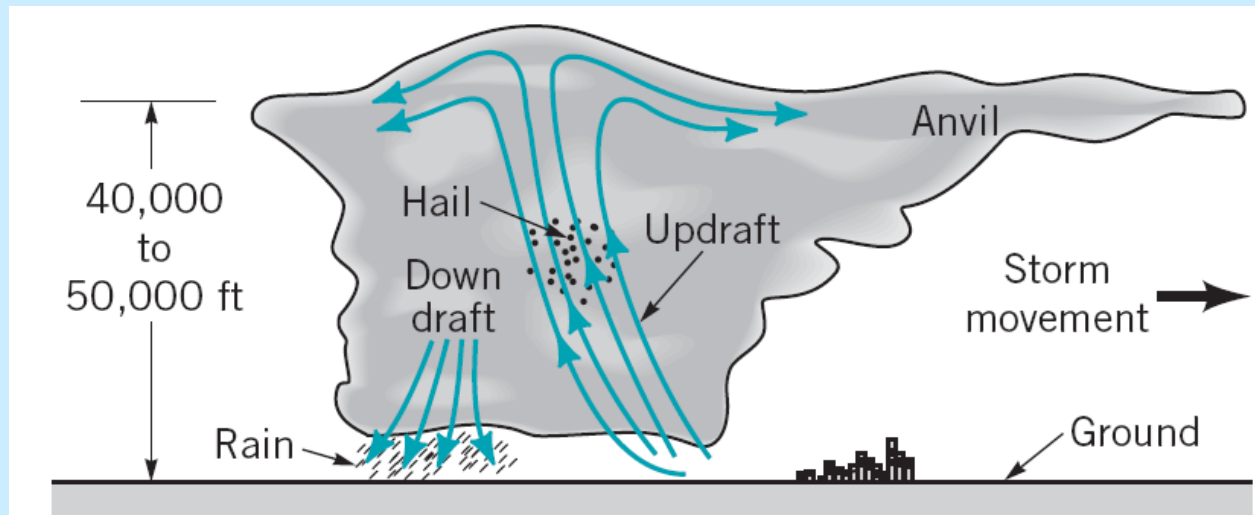# Example 9.11 Terminal velocity of a falling object

$$W = D + F_B$$

$$W = \gamma_{ice} \forall \; , \; F_B = \gamma_{air} \forall$$

$$\frac{1}{2}\rho_{air}U^2 \frac{\pi}{4}D^2 C_D = W - F_B$$

$$\gamma_{ice} \gg \gamma_{air} \quad \therefore W \gg F_B$$

$$\frac{1}{2}\rho_{air}U^2 \frac{\pi}{4}D^2 C_D = W = \gamma_{ice} \forall$$

$$U = \left(\frac{4}{3}\frac{\rho_{ice}}{\rho_{air}}\frac{gD}{C_D} \right)^{1/2}$$

40,000 to 50,000 ft

Anvil
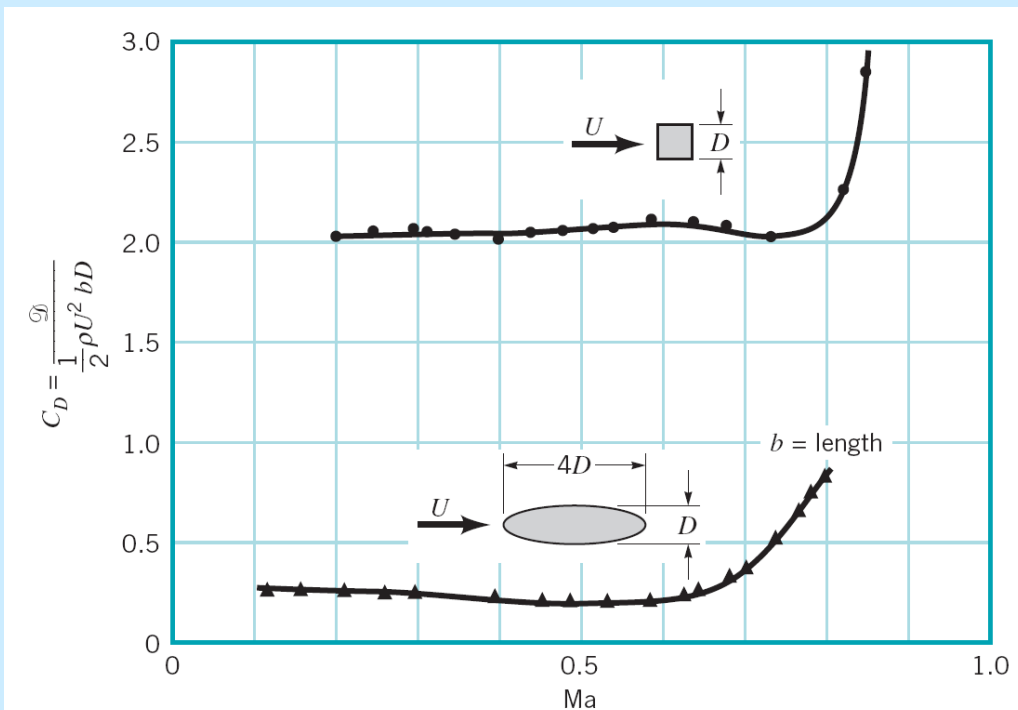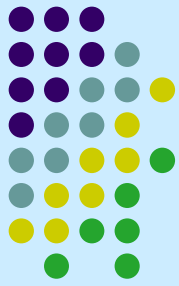
Hail

Updraft

Down draft

Storm movement

Rain

Ground

Compressibility Effects

When the compressibility effects become important

$$C_D = \phi(\text{Re, Ma})$$

Notes: 1. Compressibility effect is negligible for Ma<0.5.

2. C_D increases dramatically near Ma=1, due to the existence of shock wave.
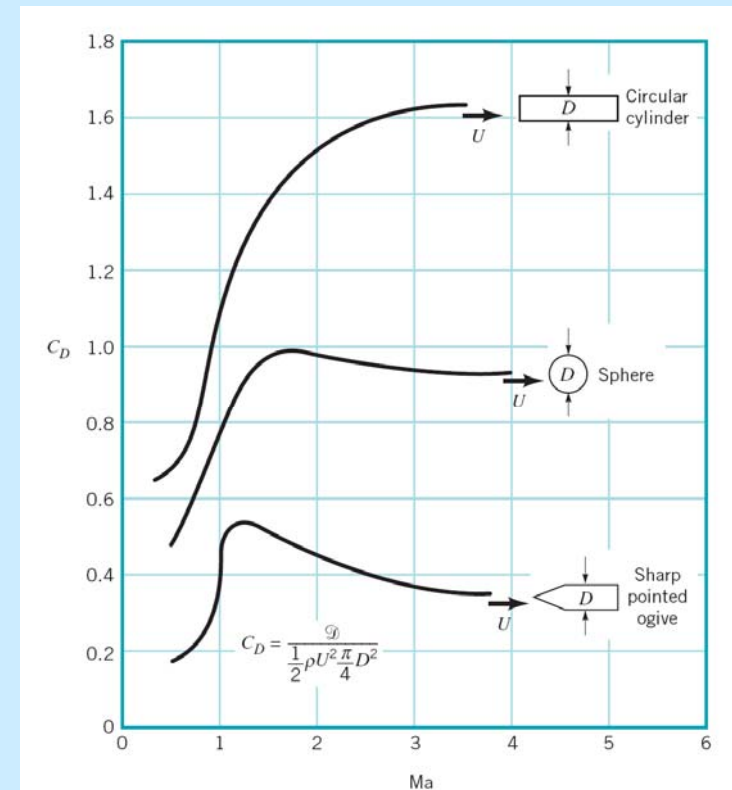
Surface Roughness Effects

- Surface roughness influences drag when the boundary layer is turbulent, because it protrudes through the laminar sublayer and alters the wall shear stress.

- In addition, surface roughness can alter the transitional critical Re and change the net drag.
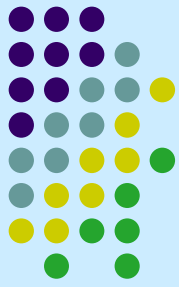
In general,

Streamlines bodies: $\dfrac{\varepsilon}{D} \uparrow \rightarrow C_D \uparrow$

Blunt bodies: $\dfrac{\varepsilon}{D} \uparrow \rightarrow C_D \square$ constant (pressure drag)

until transition to turbulence occurs and the wake region becomes considerably narrower so that the pressure drag drops. (Fig. 9.25)
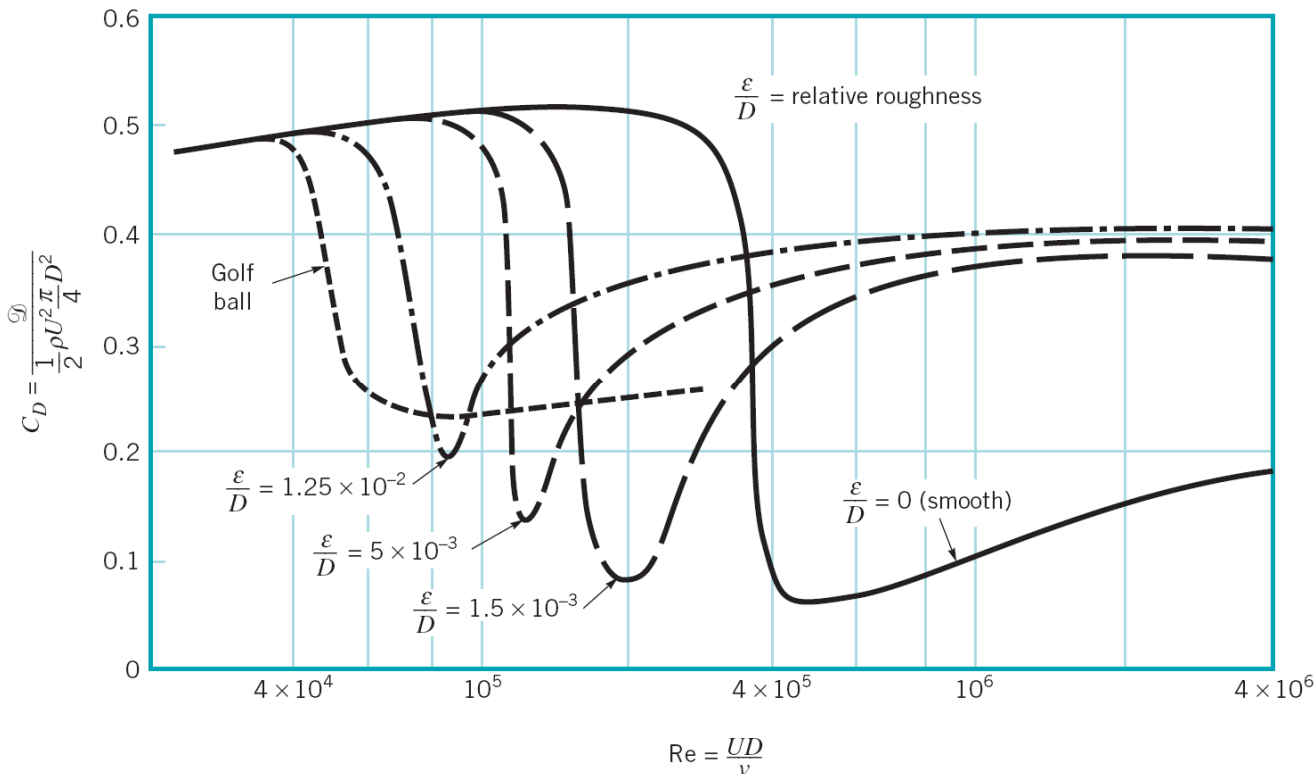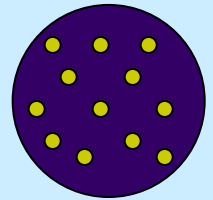
Surface Roughness

A well-hit golf ball has Re of $O(10^5)$, and the dimpled golf ball has a critical Reynolds number $4 \times 10^4 \rightarrow$ dimples reduce C_D

Table tennis Reynolds number is less than $4 \times 10^4 \rightarrow$ no need of dimples

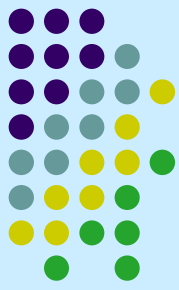**Example 9.12** Effect of Surface Roughness for golf ball and table tennis ball

dimpled

$$\frac{C_{D,\text{rough}}}{C_{D,\text{smooth}}} = \frac{0.25}{0.5} = 0.5$$

Froude Number Effects (flows with free surface)

When the free surface is present, the wave-making effects becomes important, so that
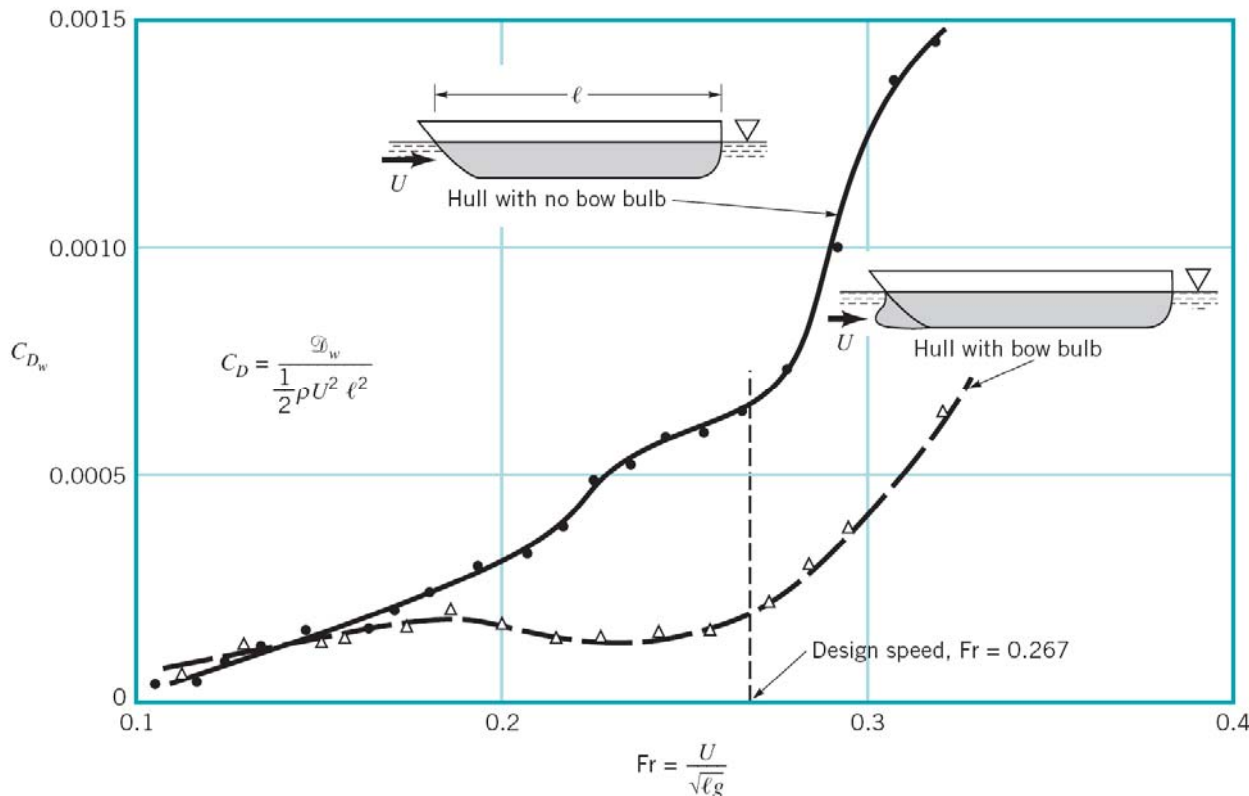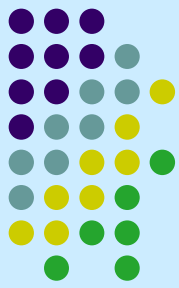
$$C_D = \phi(\text{Re, Fr})$$

Figure 9.26 (p. 507)
Typical drag coefficient data as a function of Froude number and hull characteristics for that portion of the drag due to the generation of waves.

Composite body drag

- Approximate drag calculations for a complex body by treating it as composite collection of its various parts.

  e.g., drag on an airplane or an automobile.

- The contributions of the drag due to various portions of car (i.e., front end, windshield, roof, rear end, etc.) have been determined. As a result it is possible to predict the aerodynamic drag on cars of a wide variety ouf body styles (Fig. 9.27).

V9.14 Automobile streamlining
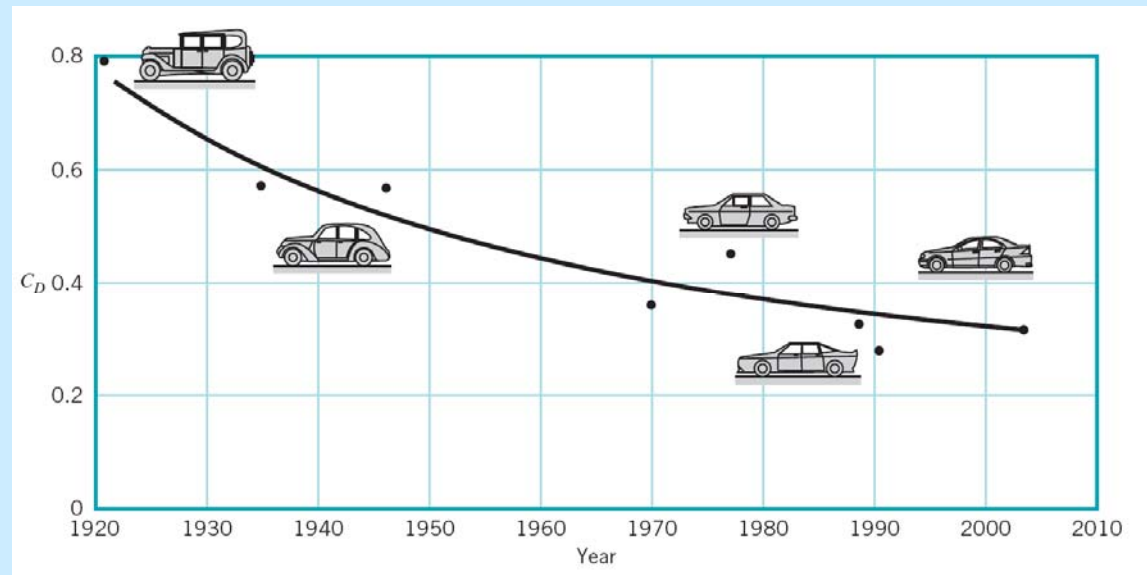
Example 9.13 Drag on a composite body

9.4 Lift (*L*)

9.4.1 Surface Pressure Distribution

- Lift -- a force that is normal to the free stream.

For aircraft, $L \uparrow$

For car, $L \downarrow$ for better traction and cornering ability

$$C_L = \frac{L}{\frac{1}{2}\rho U^2 A} \quad , \quad C_L = \phi\left(\text{shape, Re, Ma, Fr, } \frac{\varepsilon}{l}\right)$$
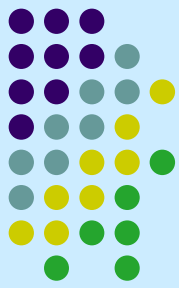
Froude no. – free surface present

ε is relatively unimportant
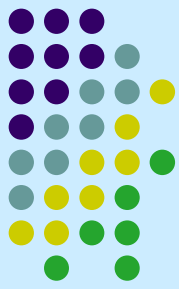
Ma is important for Ma > 0.8

Re effect is not great

The most important parameter that affects the lift coefficient is the shape of the object.

Surface Pressure Distribution

- Common lift-generating devices (airfoils, fans,…) operate at large Re. Most of the lift comes from the surface pressure distribution.

- For Re<1, viscous effect and pressure are equally important to the lift (for minute insects and the swimming of microscopic organisms).
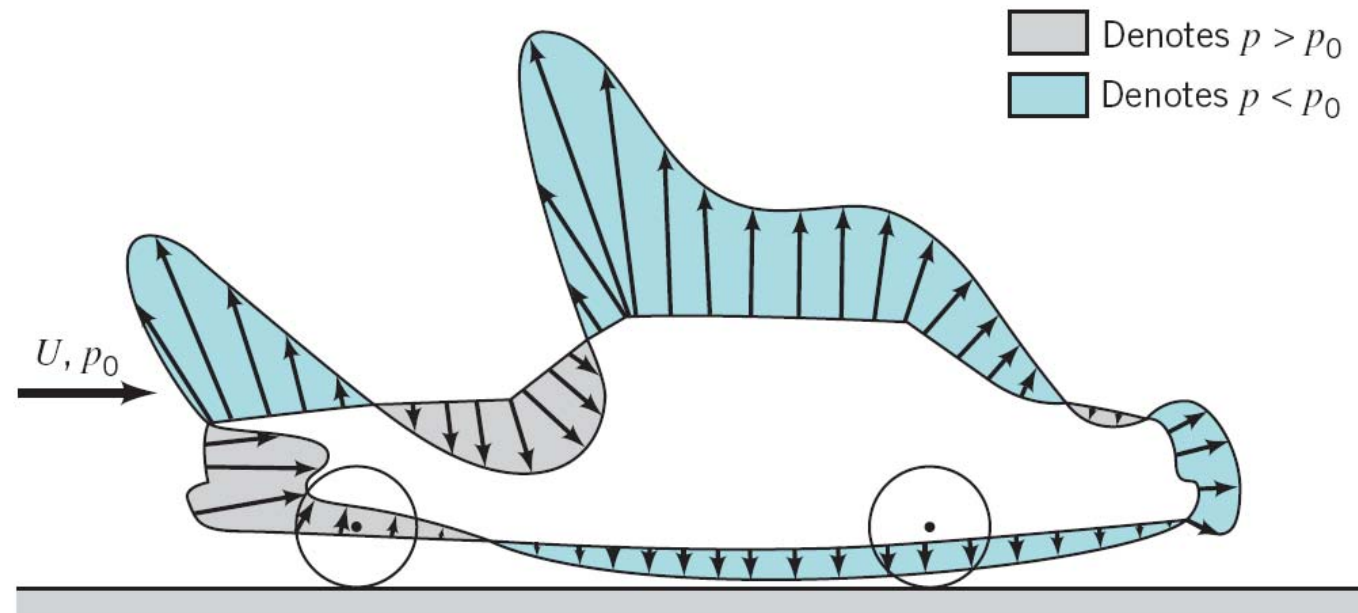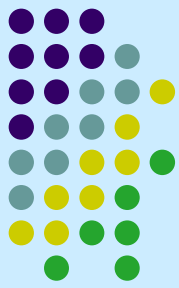
Figure 9.31

Pressure distribution on the surface of an automobile.

Denotes $p > p_0$

Denotes $p < p_0$

U, p_0

Example 9.14 Lift from pressure and shear distributions
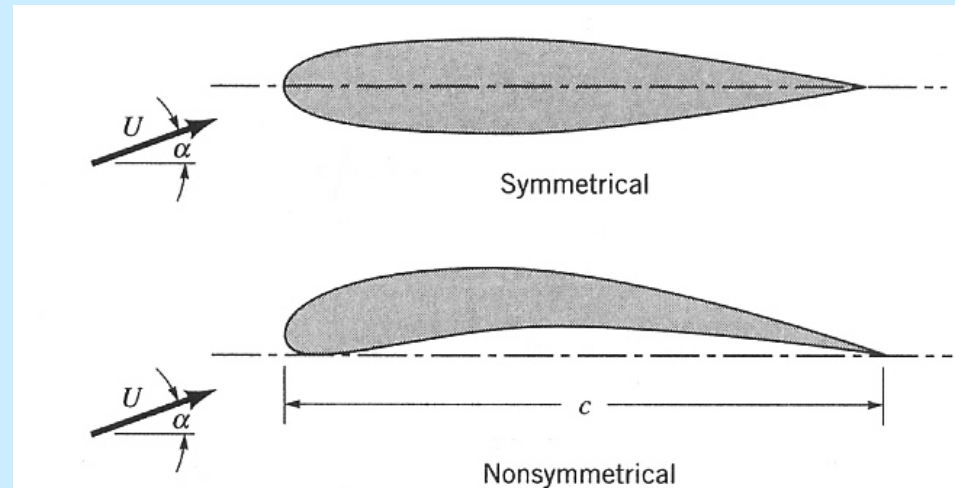
Airfoil

- For airfoils, the characteristic area A is the planform area in the definition of both C_L and C_D. For a rectangular planform wing, $A=bc$, where c is the chord length, b is the length of the airfoil.

- Typical $C_L \sim O(1)$, i.e., $L \sim (\rho U^2/2)A$, and the wing loading $L/A \sim (\rho U^2)/2$:

  Wright Flyer: $1.5 \ lb / ft^2$

  Boeing 747: $150 \ lb / ft^2$

- Aspect ratio $\mathcal{A} = b^2/A$ ($=b/c$ if c is constant)

  In general, $\mathcal{A} \uparrow \Rightarrow \ C_L \uparrow, \ C_D \downarrow$

Large $\mathcal{A}$: long wing, soaring airplane, albatross, etc.
Small $\mathcal{A}$: short wing, highly maneuverable fighter, falcon

Symmetrical

Nonsymmetrical

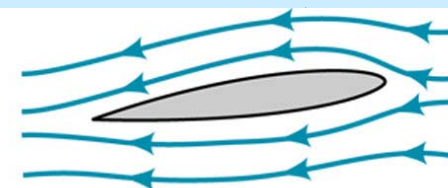# Lift and drag coefficient data as a function of angle of attack and aspect ratio
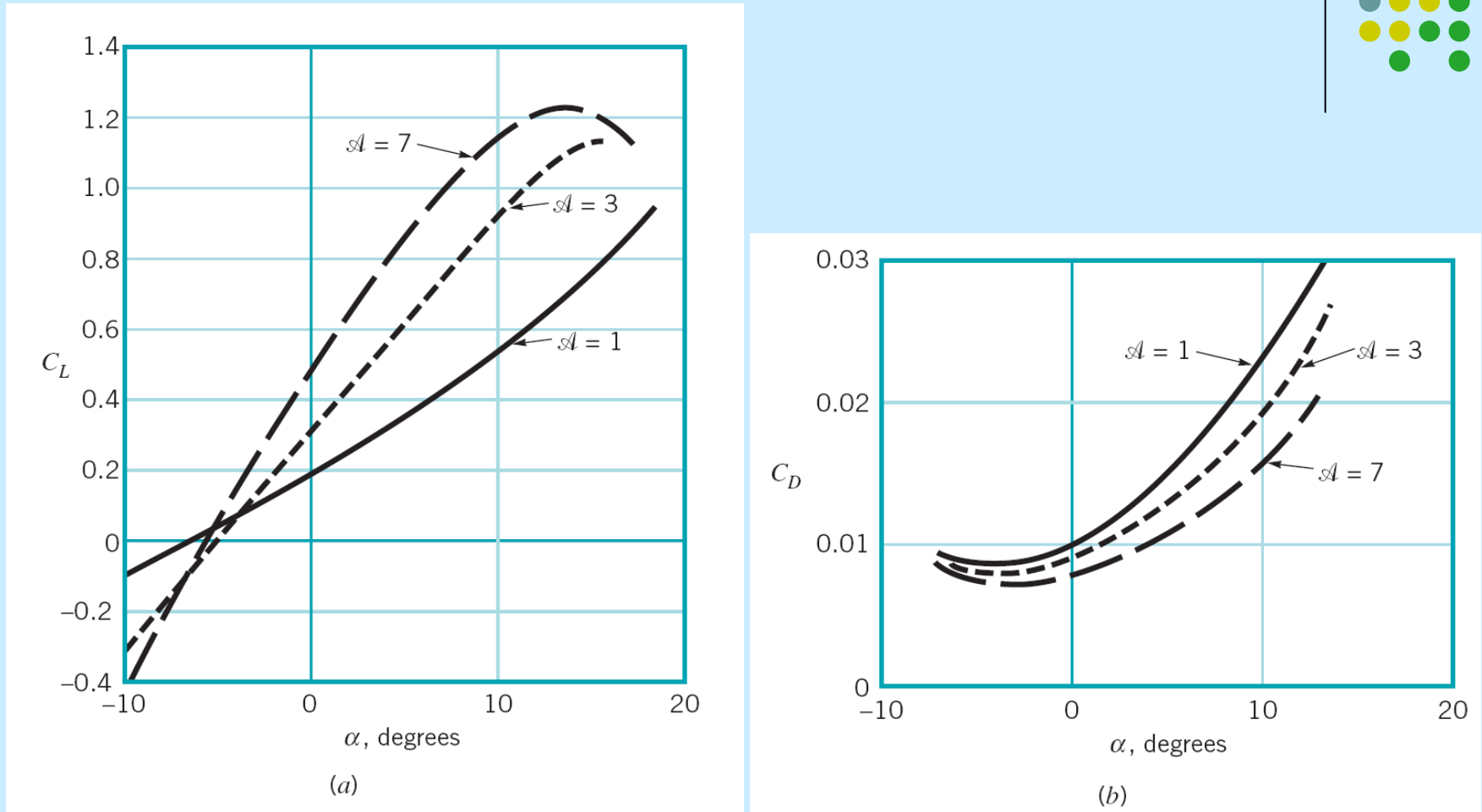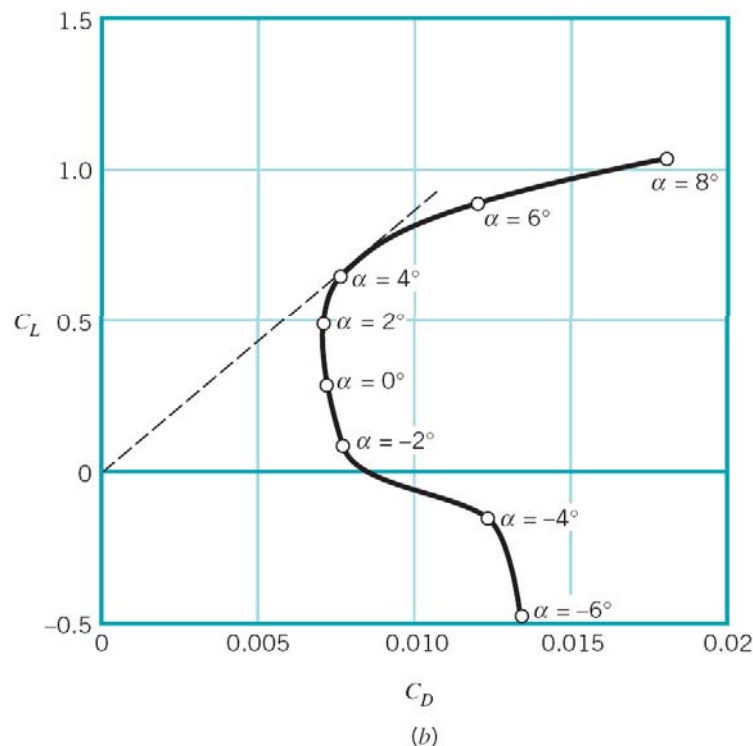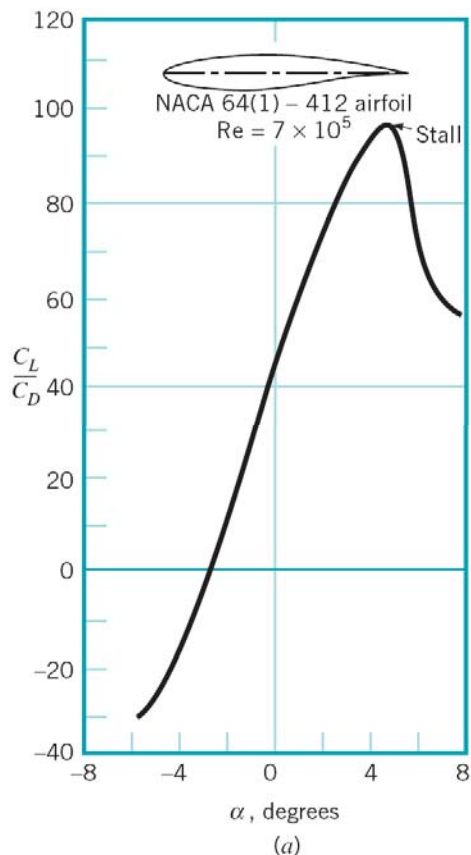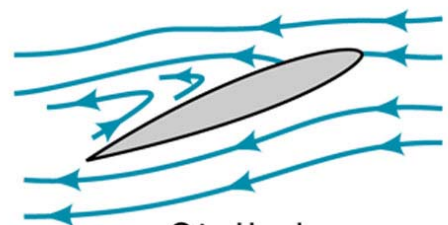

Figure 9.33
Typical lift and drag coefficient data as a function of angle of attack and the aspect ratio of the airfoil: (*a*) lift coefficient, (*b*) drag coefficient.
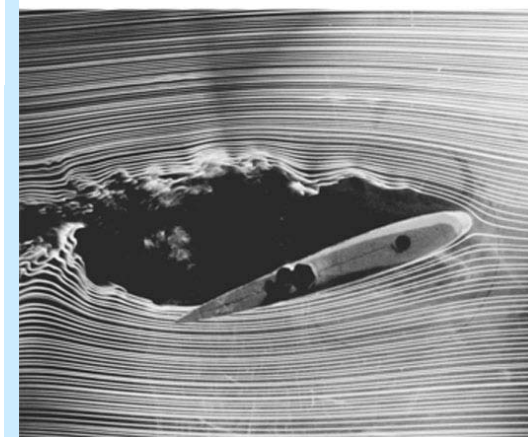
Ratio of lift to drag/Lift-drag polar

V9.15 Stalled airfoil

- Although viscous effects contributes little to the direct generation of lift, viscosity-induced boundary layer separation can occur when α is too large to lead to **stall**.

Lift of airfoil with flap

- Lift can be increased by adding flap

V9.17 Trailing edge flap

V9.18 Leading edge flap

Ex. 9.15

Figure 9.35
Typical lift and drag alterations possible with the use of various types of flap designs (Ref. 21).
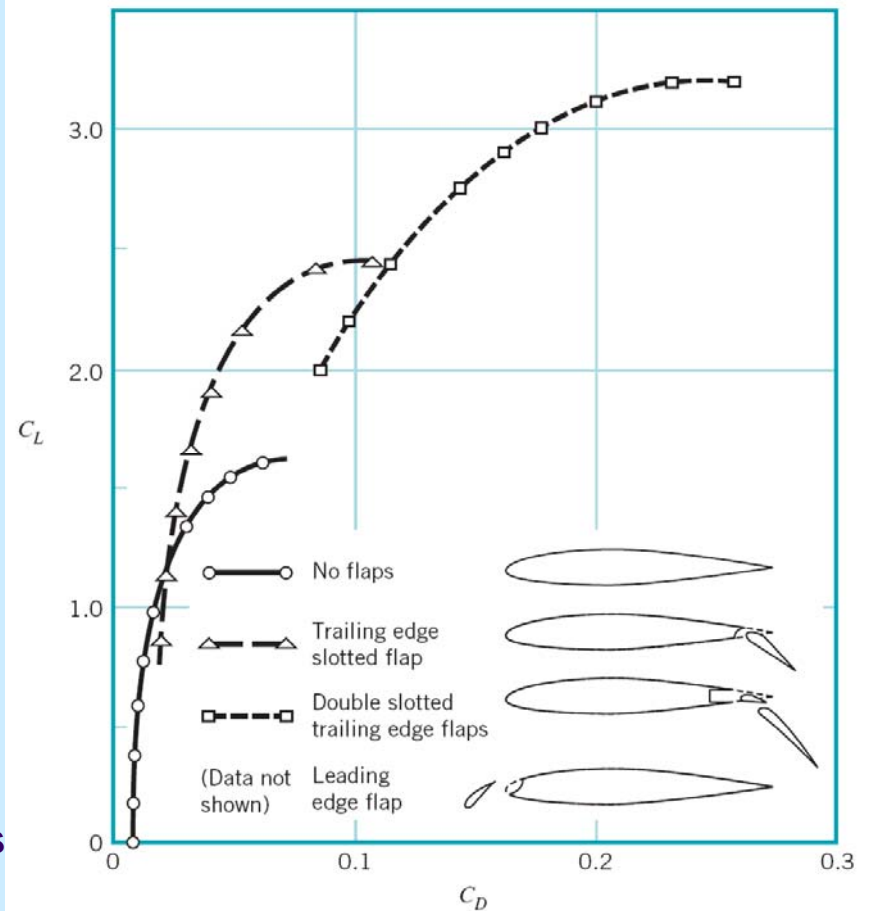

FIGURE 11–44

The lift and drag characteristics of an airfoil during takeoff and landing can be changed by changing the shape of the airfoil by the use of movable flaps.
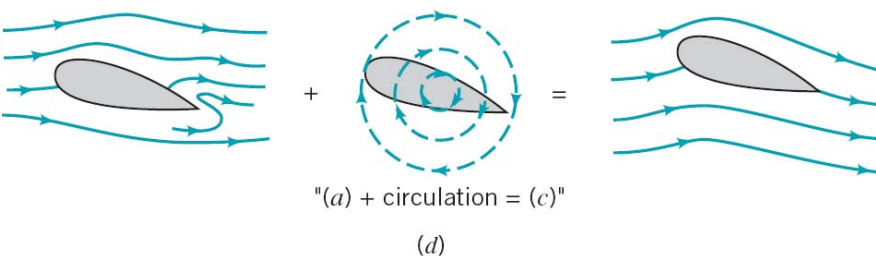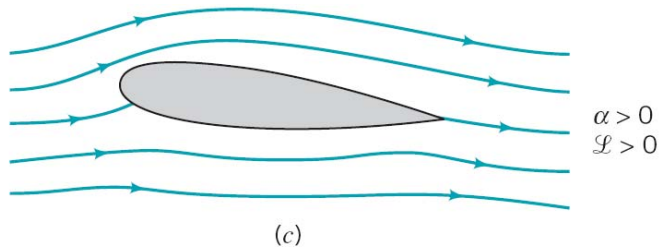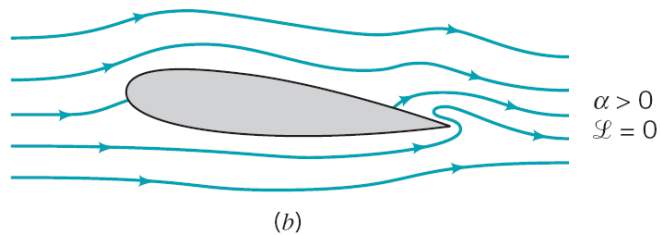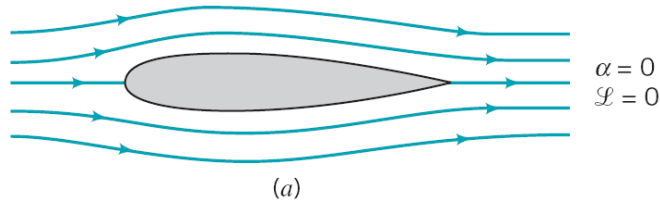
Photo by Yunus Çengel.

(a) Flaps extended (takeoff)

(b) Flaps retracted (cruising)

From Cengel and Cimbala, Fluid Mechanics, McGraw Hill, 2006

9.4.2 Circulation

2-D symmetric air foil

$\alpha = 0$
$\mathcal{L} = 0$

(a)

$\alpha > 0$
$\mathcal{L} = 0$

(b)

$\alpha > 0$
$\mathcal{L} > 0$

(c)

"(a) + circulation = (c)"

(d)
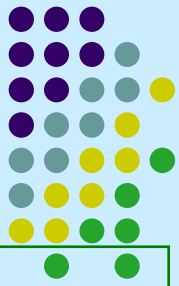
Kutta condition—The flow over both the topside and the underside join up at the trailing edge and leave the airfoil travelling parallel to one another.

--by inviscid theory

--adding circulation

Note: The circulation needed is a function of airfoil size and shape and can be calculated from potential flow theory. (Section 6.6.3)

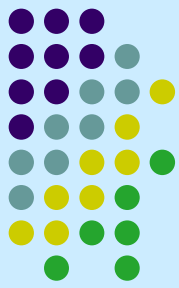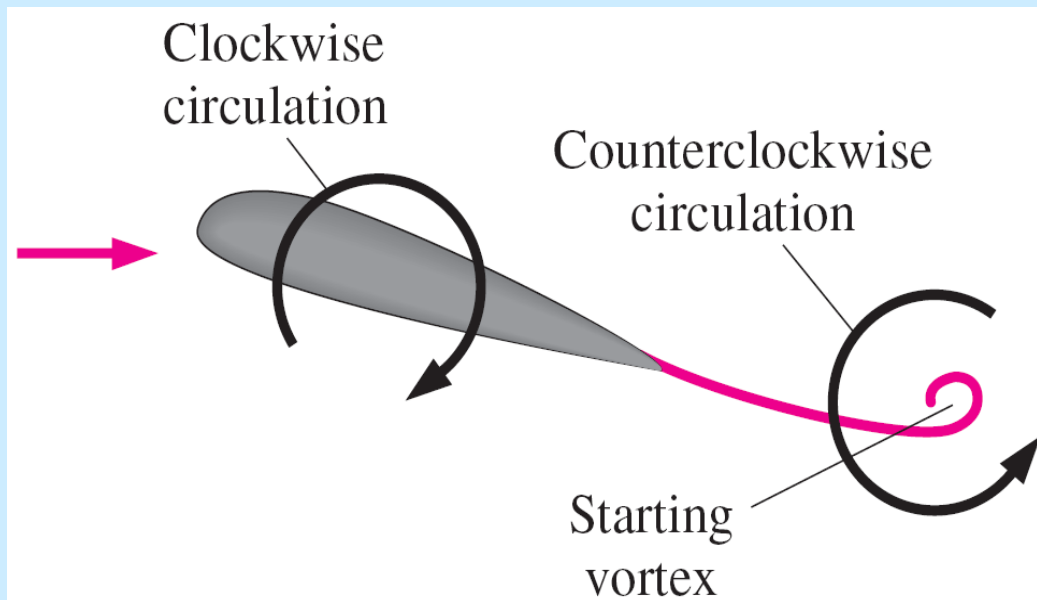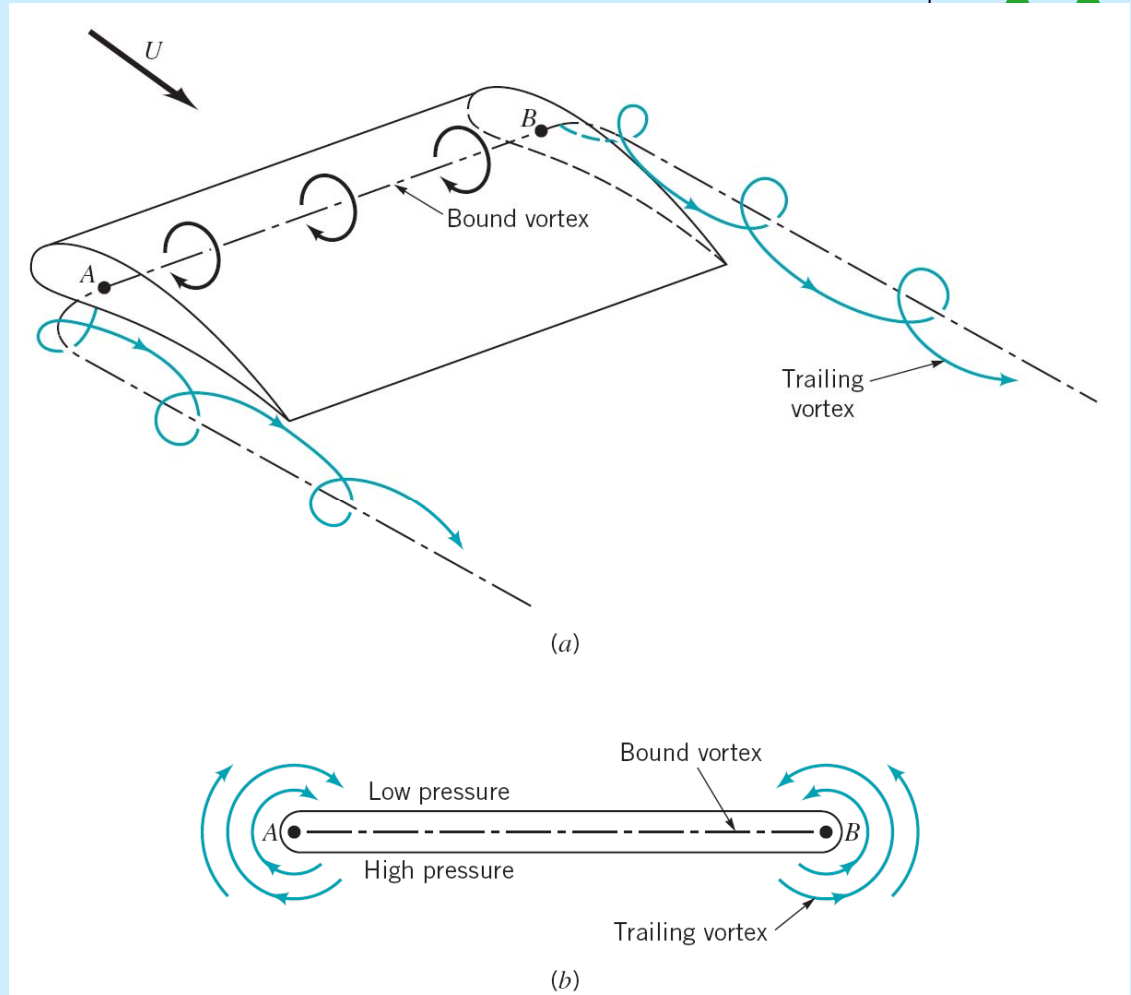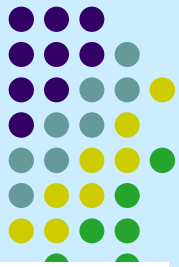$L = -\rho U \Gamma$ (Kutta–Joukowski Law)

FIGURE 11–42

Shortly after a sudden increase in angle of attack, a counterclockwise starting vortex is shed from the airfoil, while clockwise circulation appears around the airfoil, causing lift to be generated.

From Cengel and Cimbala, Fluid Mechanics, McGraw Hill, 2006

Circulation

Flow past finite length wing

Figure 9.37 (p. 546)

Flow past a finite length wing: (*a*) the horseshoe vortex system produced by the bound vortex and the trailing vortices: (*b*) the leakage of air around the wing tips produces the trailing vortices.

V4.6 Flow past a wing

V9.19 Wing tip vortex

U

Bound vortex

A

B

Trailing vortex

(*a*)

Bound vortex

Low pressure

A · — · — · — · — · — · — · — · B

High pressure

Trailing vortex

(*b*)

● Vee-formation of bird -- 25 birds fly 70% farther than 1 bird

Circulation

Flow past a circular cylinder

- A rotating cylinder in a stationary real fluid can produce circulation and generate a lift—**Magnus effect**

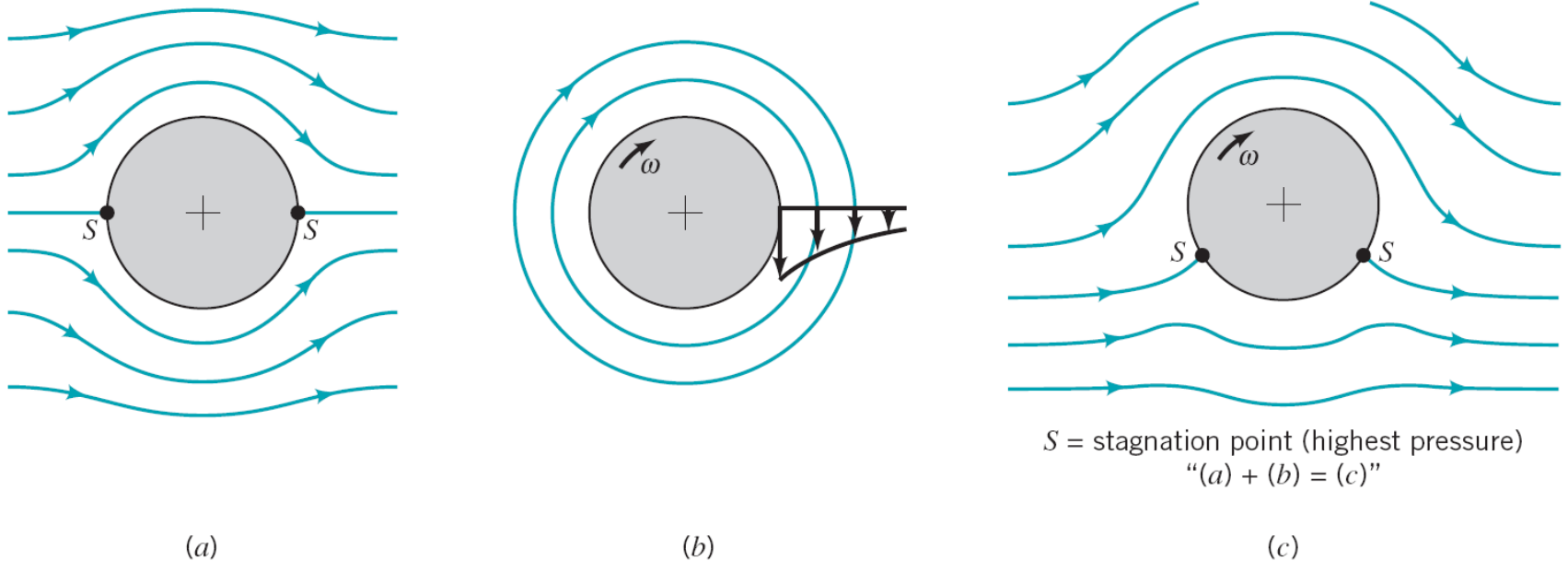**EXAMPLE 9.16** Lift on a rotating table tennis ball

S = stagnation point (highest pressure)
"$(a) + (b) = (c)$"

(a)   (b)   (c)

Figure 9.38
Inviscid flow past a circular cylinder: (*a*) uniform upstream flow without circulation. (*b*) free vortex at the center of the cylinder, (*c*) combination of free vortex and uniform flow past a circular cylinder giving nonsymmetric flow and a lift.