Common Random Numbers-Set Random Streams

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Control Random Numbers

Distributions in FLexsim

- Exponential(location, scale, stream)
- Normal(Mean Std Dev, stream)
- Beta(Min, Max, Shape 1, Shape 2, stream)
- General(xxx,xxx,xxx,...,stream)

- The last value in the parenthesis of any distribution is stream, which is called ramdom stream, used to control random numbers.

Random Numbers (亂數)

Definitions

$U_1, U_2, \ldots, U_n$ are random numbers if

- They follow uniform (0,1)
- They are independent

- random numbers are independent uniform (0,1) random variables.
- All random variables can be generated based on random numbers. (see next page)

Inverse CDF $F_X^{-1}(u)$

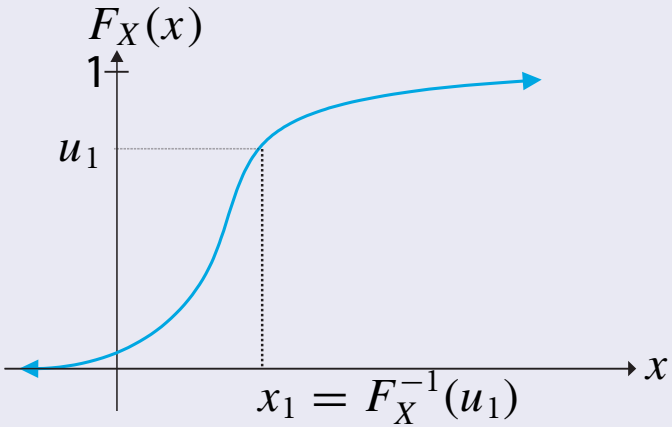
Definition

- $F_X(x) \equiv P\{X \leq x\}, \forall x \in R$
- $F_X^{-1}(u) \equiv inf\{X \in R : F_X(x) \geq u\}, u \in [0,1]$

Results

- $F_X(x) \sim U(0,1)$ for conti. X
- $U \sim U[0,1] \Rightarrow F_X^{-1}(u) \sim F_X(.)$

cdf Plot



Generate $x_1, x_2, \ldots, x_n$

- Step 1. Generate u_1 from U(0,1)
- Step 2. Obtain $x_1 = F_X^{-1}(u_1)$
- Repeat the above 2 steps n times;

  we have data: $x_1, x_2, \ldots, x_n$

Generate X based on RN

Let $U_1, U_2, \ldots, U_n$ be random numbers

Random variables

- Ex 1. $X \sim \text{Exponential}(\beta)$, $\text{E}(X) = \beta$
- Ex 2. $X \sim \text{Weibull}(\lambda, \beta)$
- Ex 3. $X \sim \text{Normal}(\mu, \sigma)$

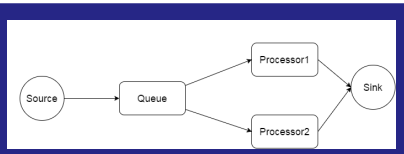
Inverse CDF

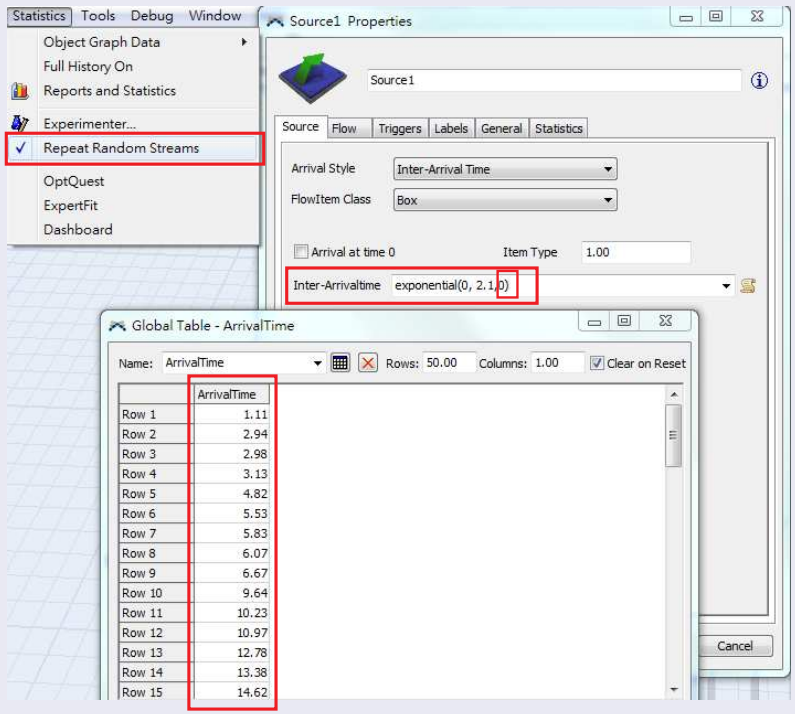
- Ex 1. $X_i = -\beta \ln(1 - U_i)$
- Ex 2. $X_i = -\beta \left[\ln(1 - U_i)\right]^{1/\alpha}$
- Ex 3. $X_i = \dfrac{(U_i^{0.135} - (1 - U_i)^{0.135})}{0.1975}$

- Use Fit-of-Goodness to test whether the above Inverse CDF algorithms work.

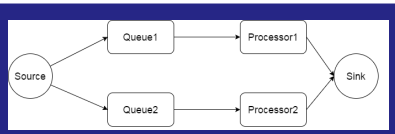
Set Random Stream = 0

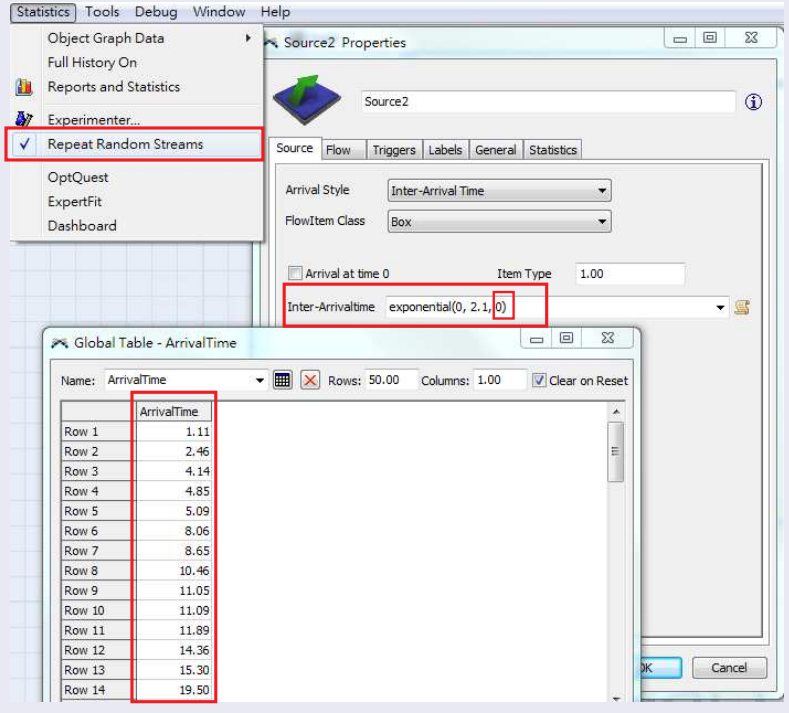
1 Queue.





2 Queues.

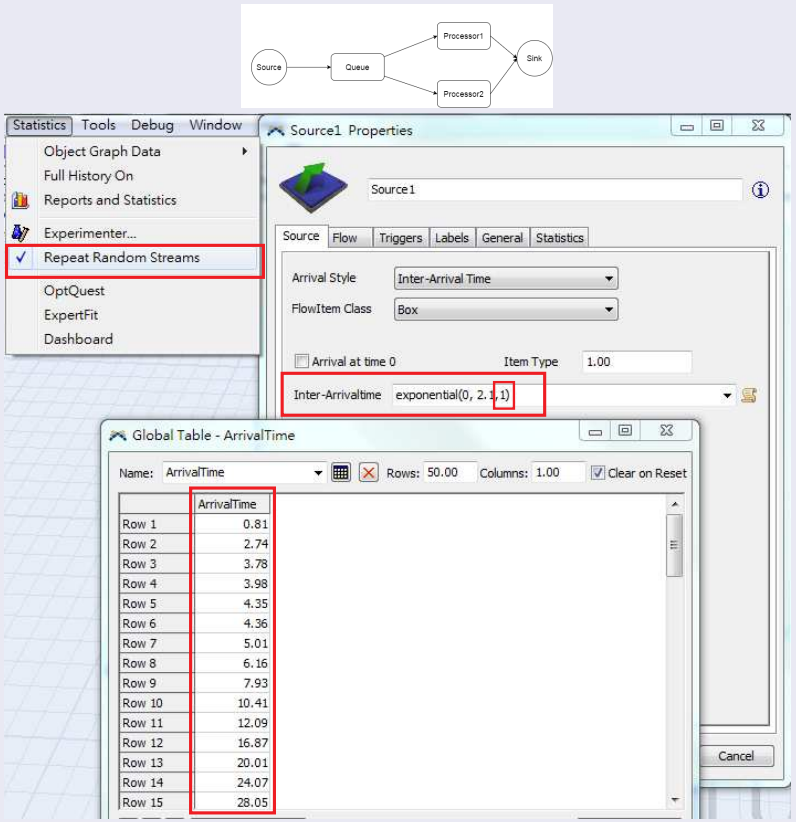




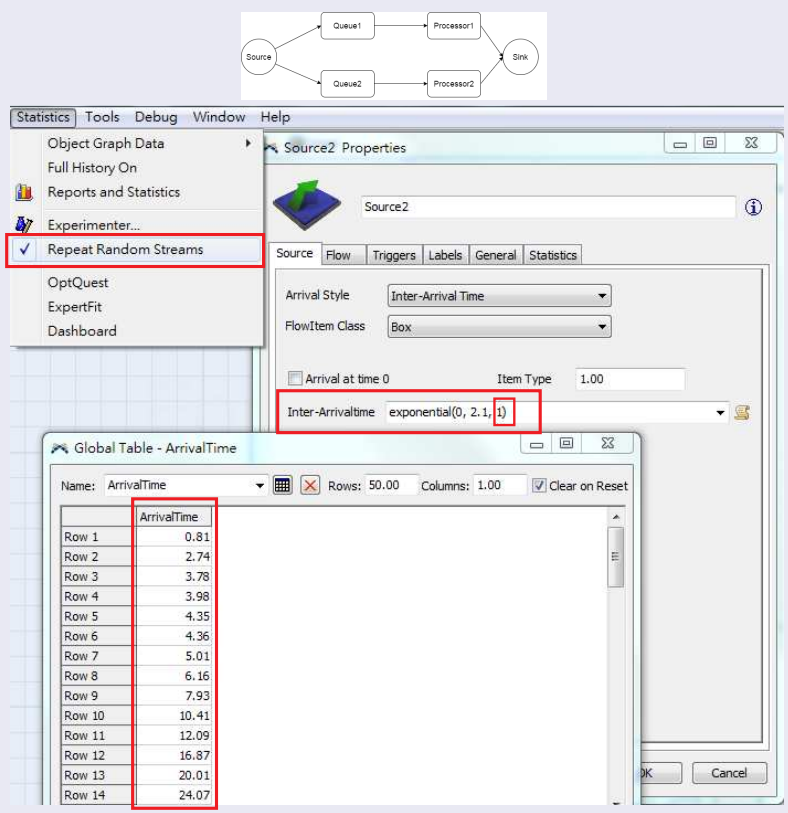
- Click “Repeat Random Streams”
- “Set random stream = 0” → Arrival Times differ

Set Random Stream = 1

System1



System2



- Click “Repeat Random Streams”
- Set “random stream $\neq 0$” $\rightarrow$ Arrival Time identical

Common Random Numbers (CRN)

- Goal: To use CRN in two systems in Flexsim

Way 1. Default distri.

- Two systems can not be in the same Flexsim file
- Click "Repeat Random Streams"
- Set the same stream value. Ex. exponential (location, scale, stream), stream value cannot be 0
- It is not easy to trace the RN

Way 2. Inverse CDF

- Use inverse cdf to generate data.
- Store the RN ($U_i, i = 1, 2, \ldots$) in Global Table
- Ex. Suppose we want to generage Exponential (mean= $1/\lambda$)
- $X_i = -ln(1 - U_i)/\lambda$, where U_i are RN
- It is easy to trace the RN

- Q: How can we use CRN to control two systems in the same flexsim file?
- A: Way 1 is not valid. We need to use Way 2