Output Analysis - Part I

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Check Correlation Plot

- Given the MM1-transient Flexsim model, name "MM1-Case1.fsm"
- Run "MM1-Case1.fsm" via "Experimenter" and view Correlation Plot
- Q: corr(WQ1, WQ2) $= 0.88$. Is this correct?

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- Q: corr(WQ2, WQ3) $= -0.8$. Is this correct?

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- A: No. corr(WQ2, WQ3) should be positive.

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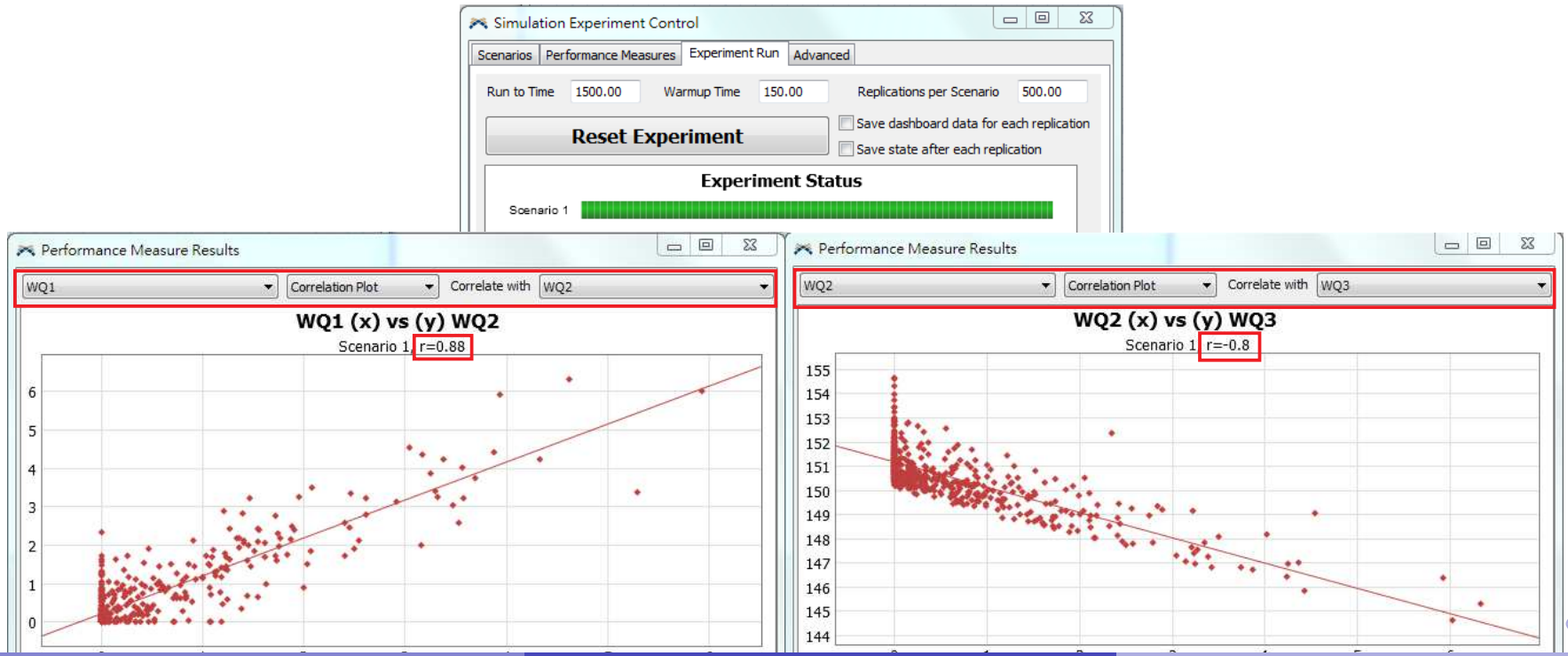
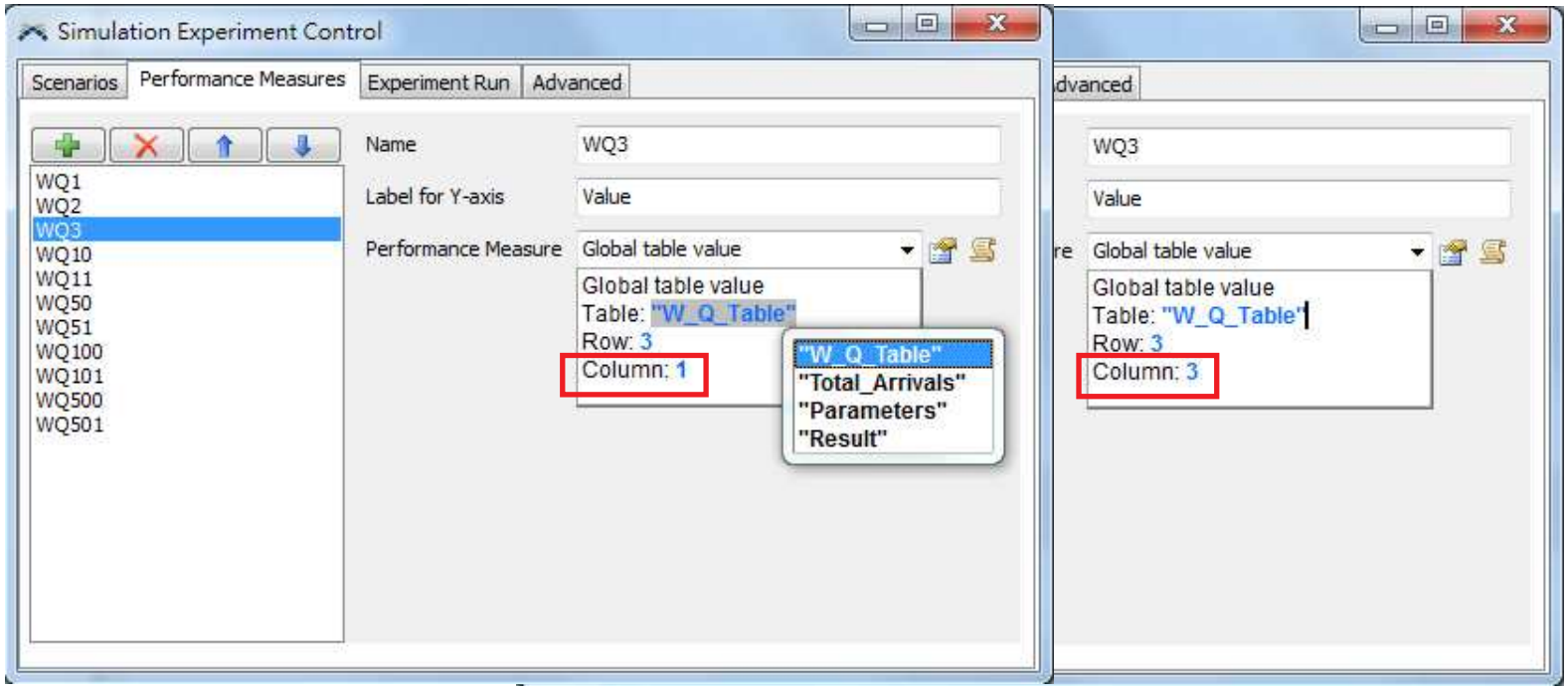


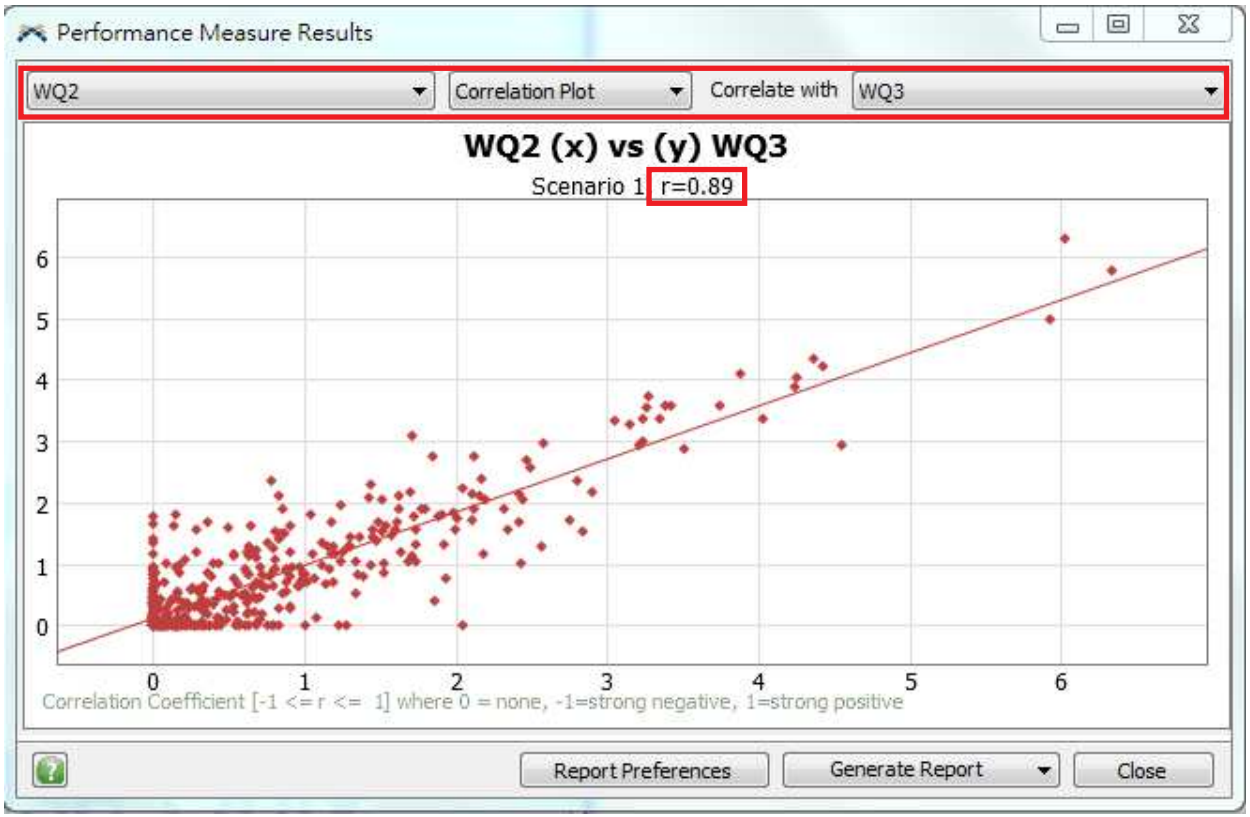
Check Correlation Plot-conti.

- Check the Performance Measures of WQ3
- Change the value of column: 1 $\rightarrow$ 3



Correct Errors - Conti.

- Correlation Plot of WQ2 and WQ3.
- corr(WQ2, WQ3)=0.89



Which WQi is in Steady State?

ID of WQi: cdf, pdf, mgf, ...

- Q: How do we fit (or estimate) the pdf of WQi?
- A: Histogram is an fitted pdf. (pp. 7-8)
- A: Relationships among pdf and the 3rd and 4th moments (p. 14)

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Property: 1st four Moments of WQi

- First 4 moments, say θ: Mean, Std, Skewness, and Kurtosis
- Let θ be the parameter and $\hat{\Theta}$ be the estimator.
- How to predict θ?
  - LDR (leading digit rule): $\hat{\Theta}$ and se($\hat{\Theta}$) (pp.10)
  - Testing Hypothesis (p.11)
  - Confidence interval (c.i.) (pp.12-13)

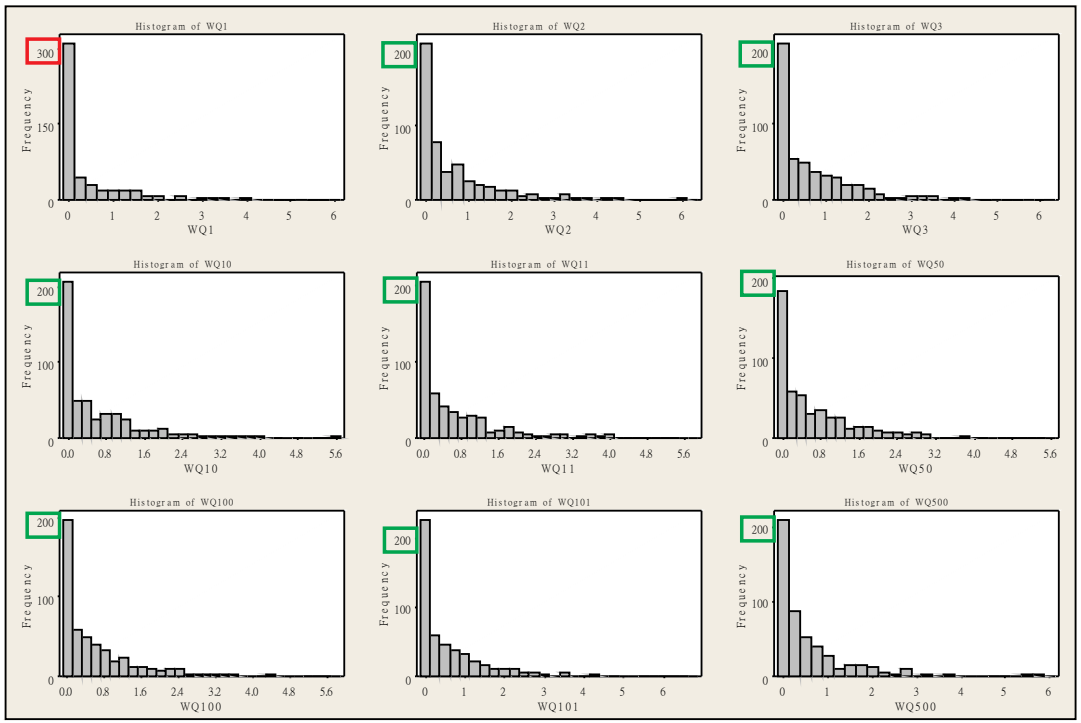
Histogram of WQi from Flexsim

- Histogram of WQ$i, i = 1, 2, 3, 10, 11, 50, 100, 101, 500$

- Q: If WQ$i, i = 1, 2, \ldots$ are in Steady State?
- A: Hard to tell via visual tools such as histogram, but it seems that WQ$i, i = 2,$ ... are in Steady State

Histogram of WQi from Minitab

- Histogram of WQi, $i = 1, 2, 3, 10, 11, 50, 100, 101, 500$ (Note: y 軸不同)



- Make plots with the same scale of y-axis
- "Histogram" is the "estimated pdf" of WQi

Results from Flexsim's

	Mean(Flexsim)	Sample Standard Deviation (Flexsim)
WQ1	0.45	0.88
WQ2	0.67	0.99
WQ3	0.70	0.96
WQ10	0.65	0.95
WQ11	0.64	0.93
WQ50	0.64	0.82
WQ51	0.59	0.80
WQ100	0.63	0.91
WQ101	0.63	0.92
WQ500	0.61	0.97
WQ501	0.63	0.97

- Q: How can you report the true mean based on the above results from Flexsim?

Estimate E(WQi) via LDR

- Results from Minitab

	Mean	SE Mean	StDev	Skewness	Kurtosis
WQ1	0.4522	0.0393	0.8779	2.75	8.73
WQ2	0.6664	0.0442	0.9892	2.37	6.96
WQ3	0.6997	0.0429	0.9600	2.13	5.82
WQ10	0.6530	0.0423	0.9452	2.18	5.54
WQ11	0.6443	0.0418	0.9374	2.15	5.28
WQ50	0.6386	0.0368	0.8221	1.81	4.13
WQ51	0.5943	0.0359	0.8020	1.97	5.08
WQ100	0.6297	0.0406	0.9086	2.25	6.09
WQ101	0.6271	0.0413	0.9228	2.34	7.28
WQ500	0.6147	0.0433	0.9680	2.75	9.46
WQ501	0.6324	0.0432	0.9669	2.80	10.25

- Q: Can we estimate first 4 moments?
- A: we can only estimate the 1st moment because no SE are given for StDev, Skewness, and Kurtosis

Testing Hypothesis

Hypothesis

$$\begin{cases} H_0 : \mathrm{E}(WQ1) - \mathrm{E}(WQ2) = 0 \\ H_1 : \mathrm{E}(WQ1) - \mathrm{E}(WQ2) \neq 0 \end{cases}$$

Two-Sample T-Test and CI: WQ1, WQ2

```
Two-sample T for WQ1 vs WQ2

      N   Mean  StDev  SE Mean
WQ1   500  0.452  0.878    0.039
WQ2   500  0.666  0.989    0.044


Difference = mu (WQ1) - mu (WQ2)
Estimate for difference:  -0.2142
95% CI for difference:  (-0.3303, -0.0981)
T-Test of difference = 0 (vs not =): T-Value = -3.62 P-Value = 0.000 DF = 998
Both use Pooled StDev = 0.9352
```

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  we claim that E(WQ1) $\neq$ E(WQ2)

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Hypothesis

$$\begin{cases} H_0 : E(WQ10) - E(WQ11) = 0 \\ H_1 : E(WQ10) - E(WQ11) \neq 0 \end{cases}$$

Two-Sample T-Test and CI: WQ11, WQ10

Two-sample T for WQ11 vs WQ10

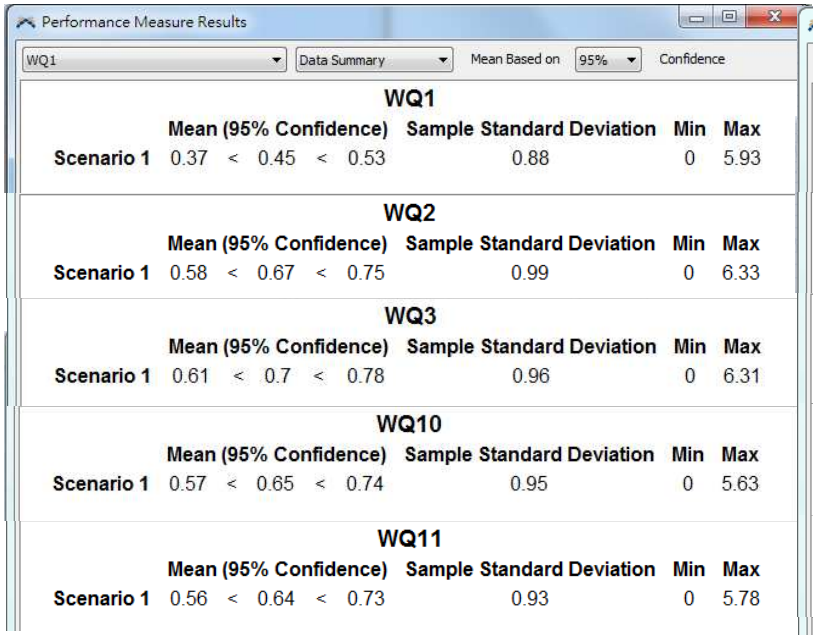
	N	Mean	StDev	SE Mean
WQ11	500	0.644	0.935	0.042
WQ10	500	0.653	0.945	0.042

Difference = mu (WQ11) - mu (WQ10)
Estimate for difference: -0.0087
95% CI for difference: (-0.1254, 0.1080)
T-Test of difference = 0 (vs not =): T-Value = -0.15 **P-Value = 0.884** DF = 998
Both use Pooled StDev = 0.9400

- Do not reject H_0

Flexsim Results: C.I. of E(WQi)

- Data Summary of WQi, $i = 1, 2, 3, 10, 11, 50, 51, 100, 101, 500, 501$


Performance Measure Results

WQ1 | Data Summary | Mean Based on 95% Confidence

WQ1

	Mean (95% Confidence)	Sample Standard Deviation	Min	Max
Scenario 1	0.37 < 0.45 < 0.53	0.88	0	5.93

WQ2

	Mean (95% Confidence)	Sample Standard Deviation	Min	Max
Scenario 1	0.58 < 0.67 < 0.75	0.99	0	6.33

WQ3

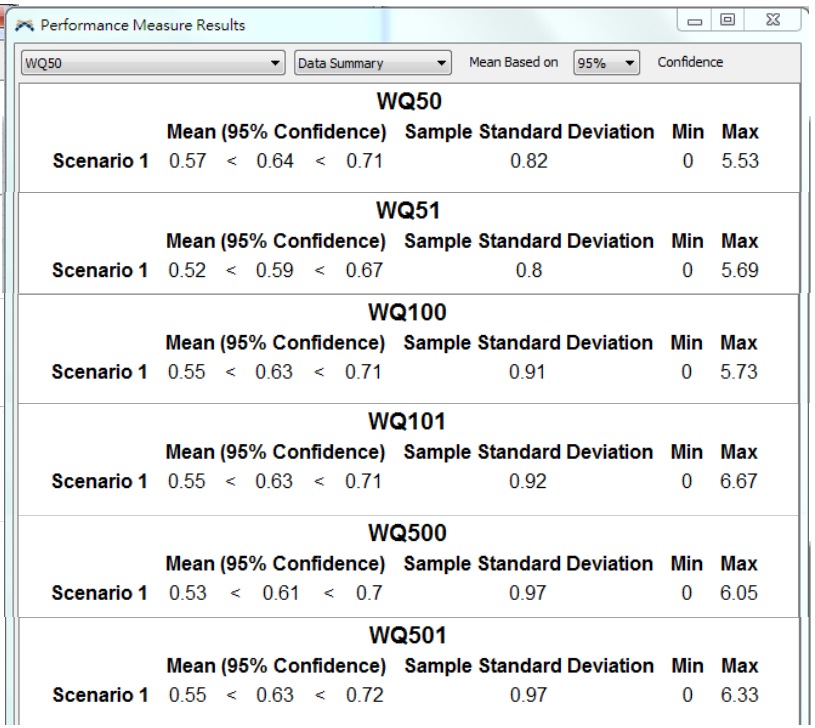
	Mean (95% Confidence)	Sample Standard Deviation	Min	Max
Scenario 1	0.61 < 0.7 < 0.78	0.96	0	6.31

WQ10

	Mean (95% Confidence)	Sample Standard Deviation	Min	Max
Scenario 1	0.57 < 0.65 < 0.74	0.95	0	5.63

WQ11

	Mean (95% Confidence)	Sample Standard Deviation	Min	Max
Scenario 1	0.56 < 0.64 < 0.73	0.93	0	5.78


Performance Measure Results

WQ50 | Data Summary | Mean Based on 95% Confidence

WQ50

	Mean (95% Confidence)	Sample Standard Deviation	Min	Max
Scenario 1	0.57 < 0.64 < 0.71	0.82	0	5.53

WQ51

	Mean (95% Confidence)	Sample Standard Deviation	Min	Max
Scenario 1	0.52 < 0.59 < 0.67	0.8	0	5.69

WQ100

	Mean (95% Confidence)	Sample Standard Deviation	Min	Max
Scenario 1	0.55 < 0.63 < 0.71	0.91	0	5.73

WQ101

	Mean (95% Confidence)	Sample Standard Deviation	Min	Max
Scenario 1	0.55 < 0.63 < 0.71	0.92	0	6.67

WQ500

	Mean (95% Confidence)	Sample Standard Deviation	Min	Max
Scenario 1	0.53 < 0.61 < 0.7	0.97	0	6.05

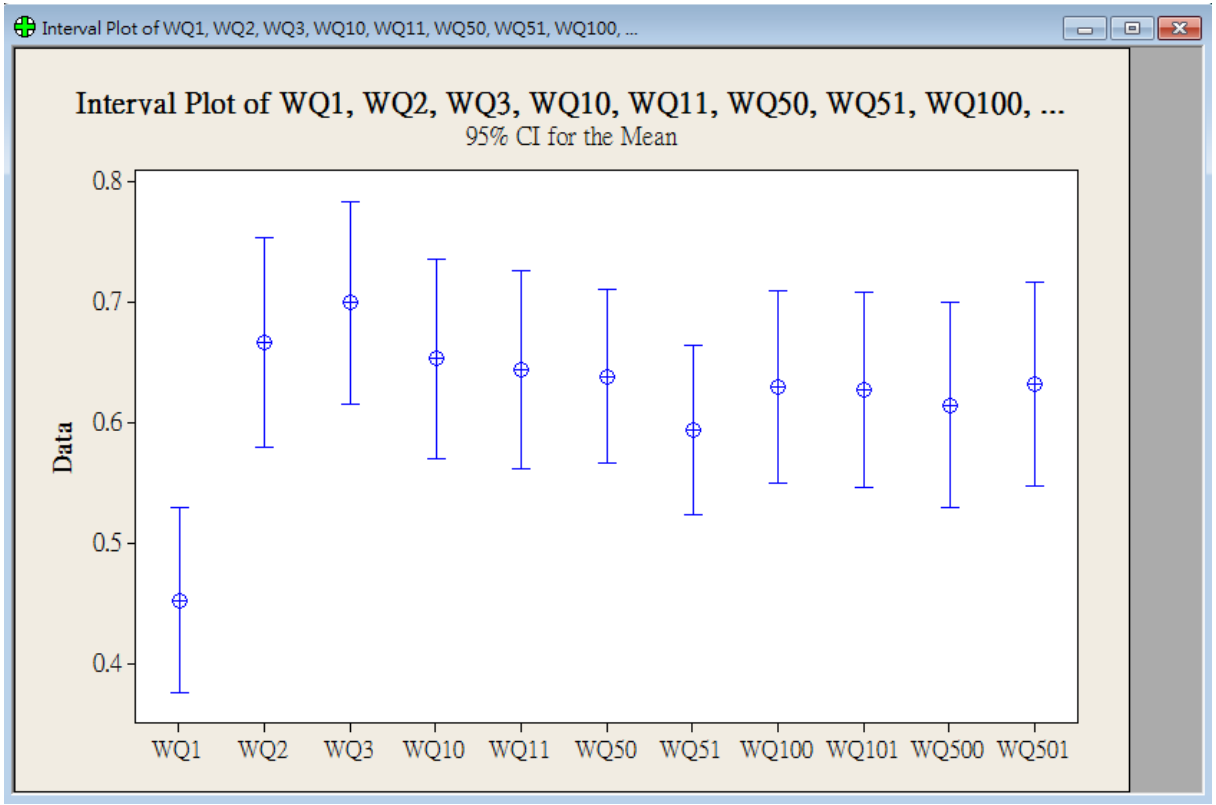
WQ501

	Mean (95% Confidence)	Sample Standard Deviation	Min	Max
Scenario 1	0.55 < 0.63 < 0.72	0.97	0	6.33

- Q: If WQi, $i = 1, 2, \ldots$ have the same first moment?
- A: Hard to tell from the above summary

Confidence Interval (Minitab)

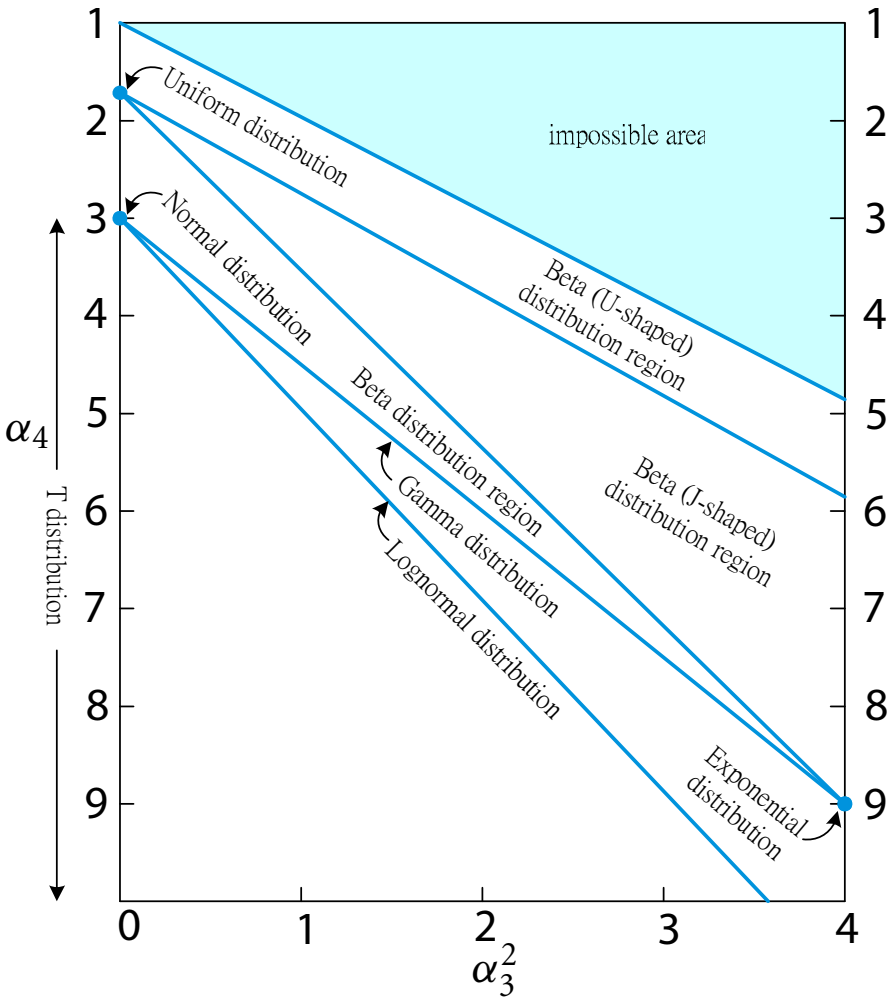
- Minitab Interval Plot of WQi, $i = 1, 2, 3, 10, 11, 50, 51, 100, 101, 500, 501$



- It seems that WQ1 is in transient.
- Recall that we delete the first 150 minutes.

Pdf vs. skewness and kurtosis

- $\alpha_3 = \mathrm{E}(X - \mu)^3/\sigma^3, \alpha_4 = \mathrm{E}(X - \mu)^4/\sigma^4$



Practice

- Distinguish θ, $\hat{\Theta}$, and $\hat{\theta}$. Relate them with today's lecture.
- Use Excel to check Flexsim Experiment Data Summary (hint: see the attached excel below)

	A	B	C	D	E	F	G	H	I
1	xbar(3600 min), rep: 50	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5			
2			(1) self-coded						
3	Mean(X)	1.0004	0.9892	0.5256	0.67	1.9784	"B3=AVERAGE(B20:B69)"		
4	sd(X)	0.06712644	0.047888	0.031633	0.024661	0.134777	"B4=STDEV(B20:B69)"		
5	se(Xbar)	0.00949311	0.006772	0.004474	0.003488	0.01906	"B5=B4/SQRT(50)"		
6	90% upper ci	1.01601617	1.00034	0.532959	0.675737	2.009754	"B6=B3+(1.645*B5)"		
7	90% lower ci	0.98478383	0.97806	0.518241	0.664263	1.947046	"B7=B3-(1.645*B5)"		
8	min	0.86	0.92	0.46	0.62	1.74	"B8=MIN(B20:B69)"		
9	max	1.16	1.12	0.6	0.73	2.35	"B9=MAX(B20:B69)"		
10			(2) flexsim provided (小數點後兩位)						
11	Mean(X)	1	0.99	0.53	0.67	1.98			
12	sd(X)	0.07	0.05	0.03	0.02	0.13			
13	se(Xbar)								
14	90% upper ci	1.02	1	0.53	0.68	2.01			
15	90% lower ci	0.98	0.98	0.52	0.66	1.95			
16	min	0.86	0.92	0.46	0.62	1.74			
17	max	1.16	1.12	0.6	0.73	2.35			
18									
19	Replication No.	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5			
20	1	1.05	0.94	0.51	0.66	1.87			
21	2	0.96	1.06	0.57	0.72	2.25			
22	3	1.02	1.01	0.53	0.67	1.98			
			⋮						
67	48	0.9	0.94	0.49	0.68	1.94			
68	49	0.99	1.04	0.53	0.65	2.09			
69	50	1.02	1.12	0.54	0.7	2.35			