Day 3: M/M/1 Model: Analytical vs. Simulation

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Introduction of FlexSim

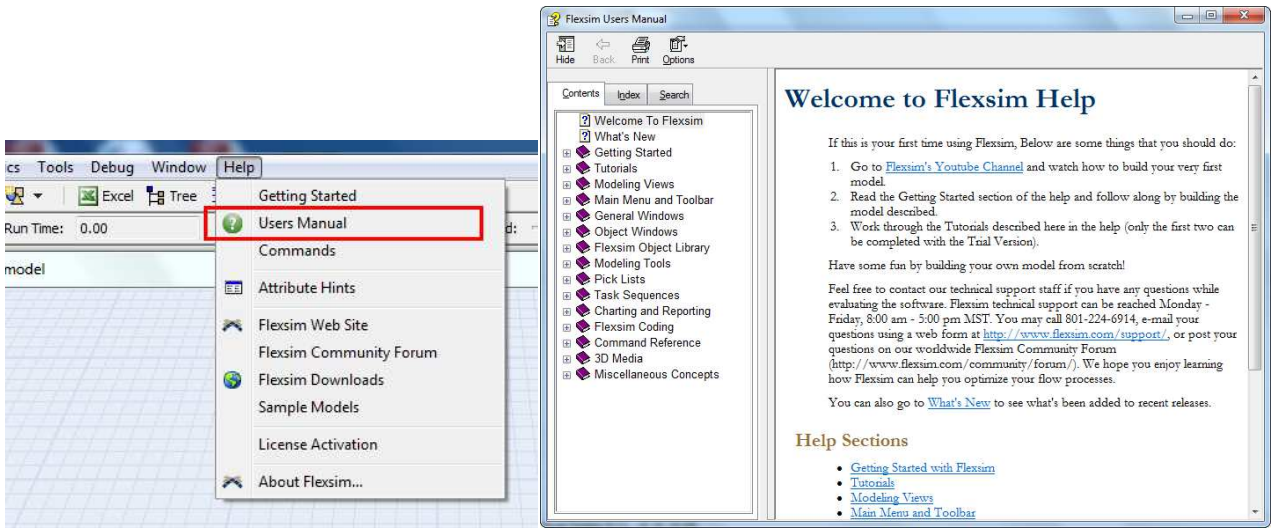
- FlexSim is a discrete event simulation software developed by FlexSim Software Products, Inc.
- FlexSim 1.0 was released in February 2003.
- The most recent of FlexSim release is version 7
- FlexSim Health Care was later developed for healthcare simulation.
- The FlexSim Software Products, Inc. headquarters is located in Orem, Utah, U.S.A.
- Trial Version "http://www.flexsim.com/" (Note: Less than 20 objects, Random seed not available)
- Education Use only: help¿licence activaction¿licence service¿ use concurrent licennsing ¿ 140.114.53.5
- What is discrete event simulation?

Flexsim Learning Platforms

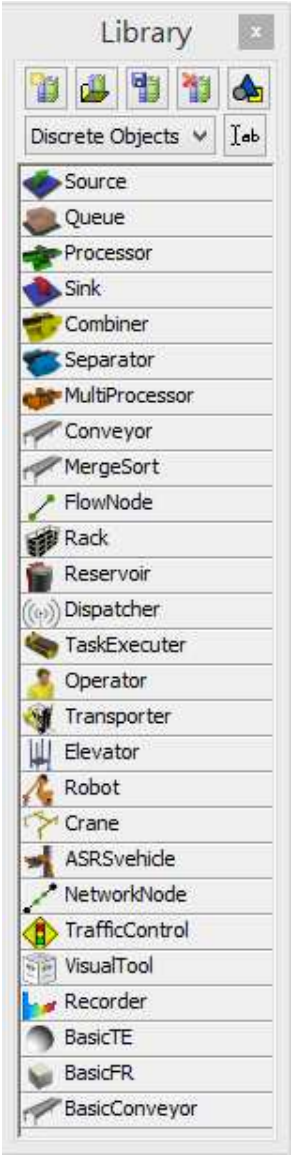
- Flexsim in USA

  `http://www.flexsim.com/community/forum/index.php`
- Flexsim in Asia (簡體字)

  `http://www.flexsim.asia/`
- FlexSim Users Manual in the toolbar

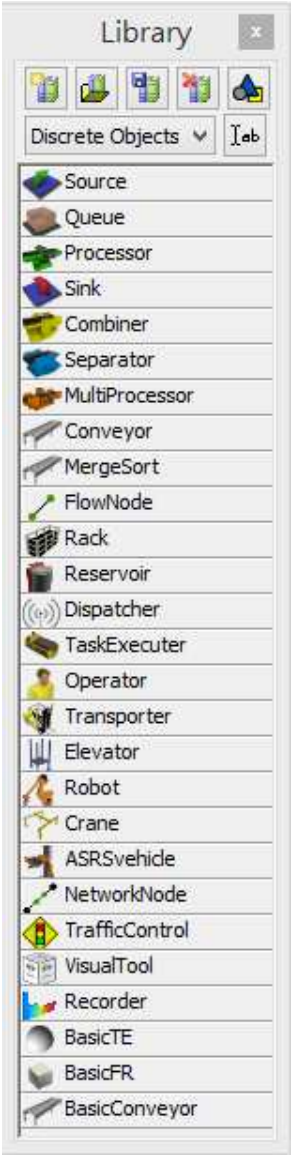


FlexSim Discrete Objects - 1



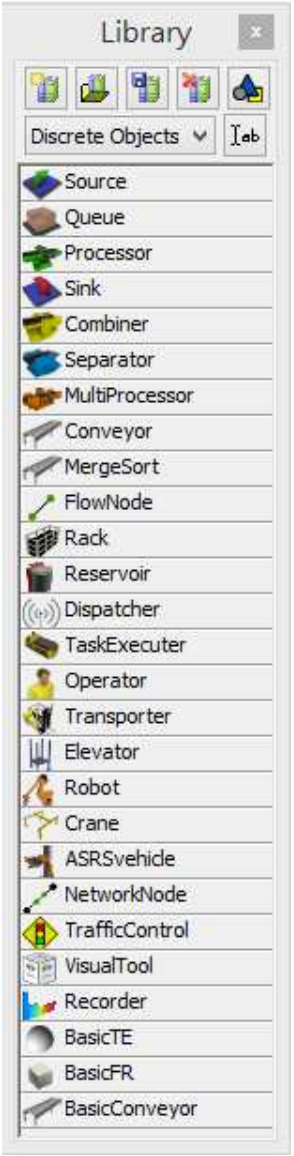
- Source: create the flowitems
- Queue: store flowitems
- Processor: process flowitems
- Sink: destroy flowitems

FlexSim Discrete Objects - 2



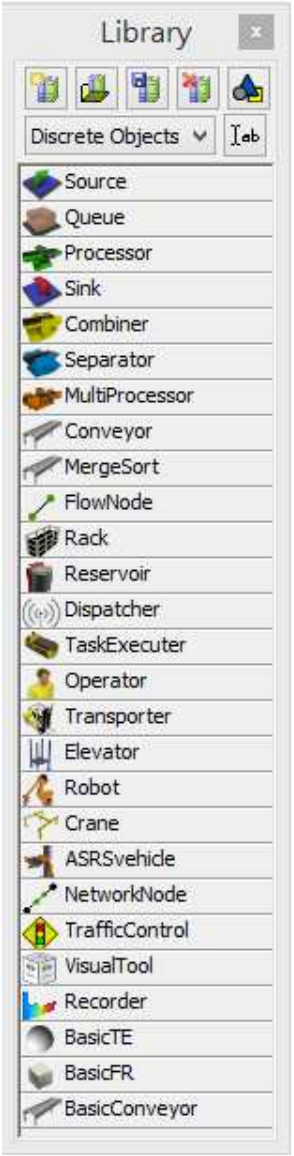
- Combiner: group multiple flowitems together
- Separator: separate a flowitem into multiple parts
- MultiProcessor: simulate the processing of flowitems in sequentially ordered operations
- Conveyor: move flowitems along a set path
- MergeSort: non-accumulating conveyor that allowsto have multiple input positions and multiple output positions along the conveyor
- FlowNode: move flowitems from one location to another with time being consumed

FlexSim Discrete Objects - 3



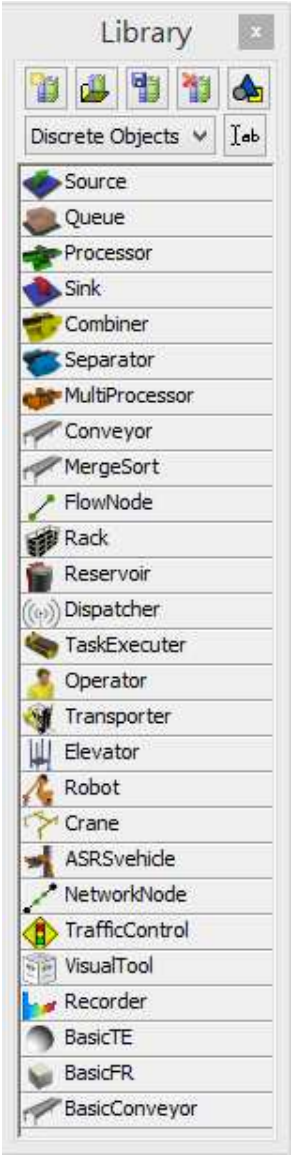
- Rack: store flowitems as if they were in a warehouse rack
- Reservoir: store flowitems as if they were in a fluid reservoir or tank
- Dispatcher: control a group of transporters or operators
- TaskExecuter: the top level class for Operators, Transporters, ASRSvehicles, Cranes and other mobile resources
- Operator: can be called by objects to be utilized during setup, processing or repair time

FlexSim Discrete Objects - 4



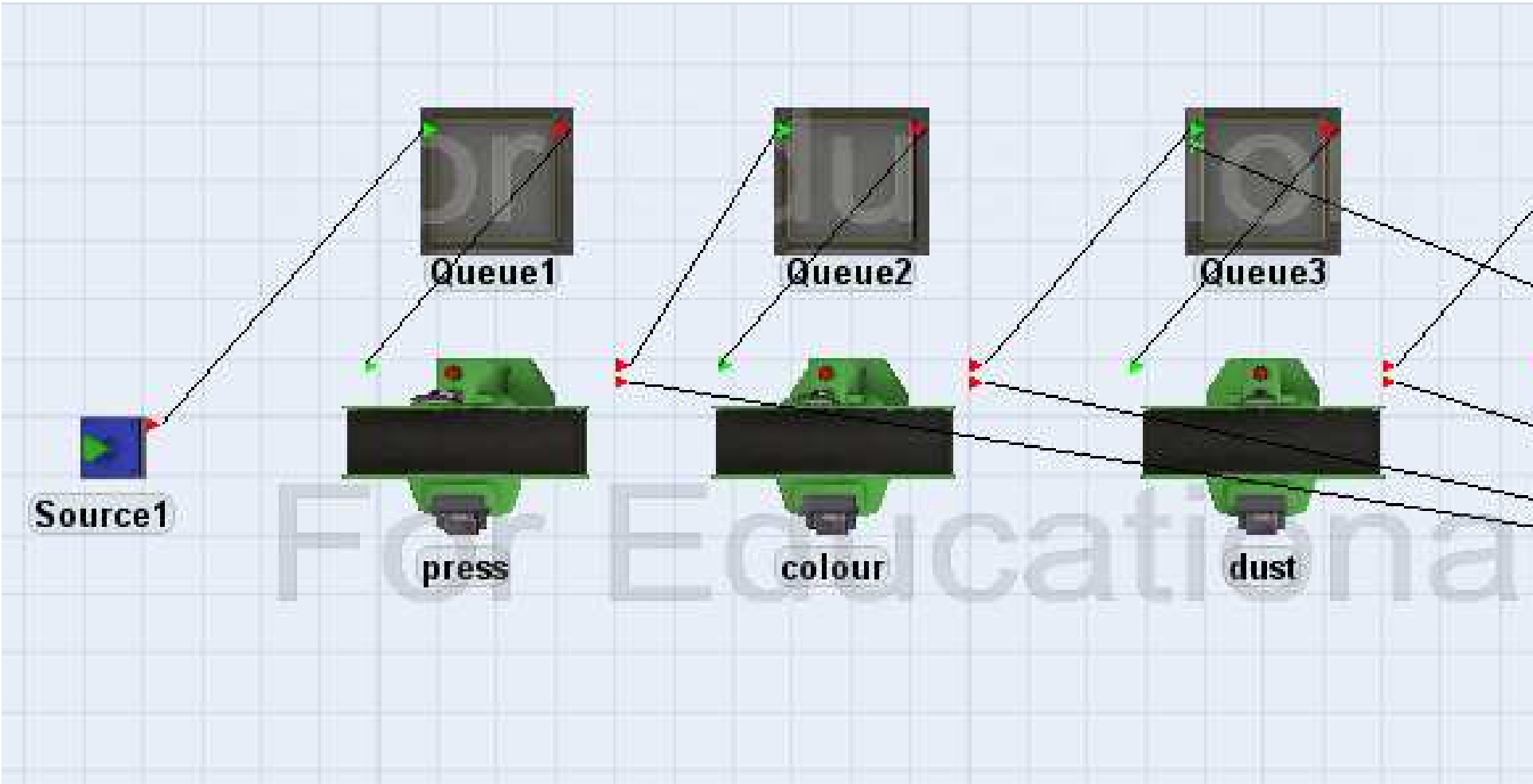
- Transporter: used mainly to carry flowitems from one object to another
- Elevator: a special type of transport that moves flowitems up and down
- Robot: a special transport that lifts flowitems from their starting locations and places them at their ending locations
- Crane: similar functionality to the transporter but with a modified graphic
- ASRSvehicle: a special type of transport specifically designed to work with racks
- NetworkNode: define a network of paths that transporters and operators follow

FlexSim Discrete Objects - 5



- TrafficControl: control traffic in a given area of a travel network
- VisualTool: used to decorate the model space with props, scenery, text, and presentation slides in order to give the model a more realistic appearance
- Recorder: record and/or display information graphically
- BasicTE: a TaskExecuter that is meant for developers to create user libraries with
- BasicFR: a FixedResource that is designed to be customized into a user library object
- BasicConveyor: a conveyor that allows flowitems to move along the conveyor according to logic that is defined by the user

Model: M/M/1



Notations

- λ, μ: arrival rate, service rate
- P_n: P(n customers in system)
- L: Length (no. of people wait) in system
- L_q: Length in queue
- W: wait time in system
- W_q: wait time in queue
- $\rho = \frac{\lambda}{\mu}$,
- Littles' formula:
  $E(L) = \lambda E(W)$
  $E(L_q) = \lambda E(W_q)$
- which one is random variable?

M/M/1 Analytical Solution

$$P_o = 1 - \rho$$

$$P_n = \rho^n \cdot P_0$$

$$E(L) = \sum_{n=0}^{\infty} n \cdot P_n = \frac{\rho}{1-\rho}$$

$$E(L_q) = E(L) - (1 - P_o)$$

$$E(W) = E(L)/\lambda$$

$$E(W_q) = E(L_q)/\lambda$$

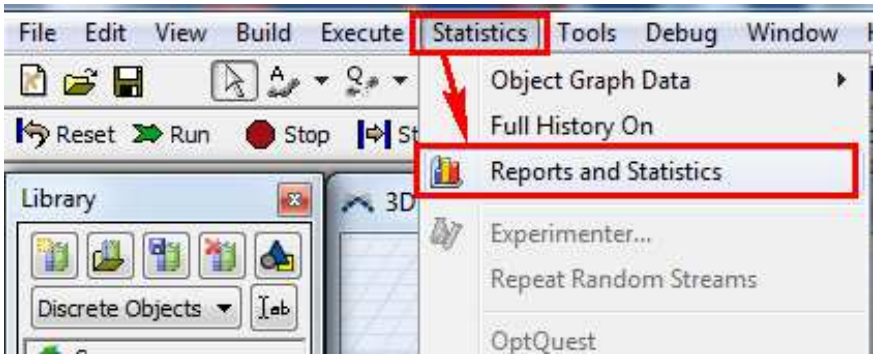
- Illustrate the meaning of the above notations

Simulate M/M/1

- Open Flexsim 6
- Build new model
- Repeat Random Streams to guarantee Common Random Numbers

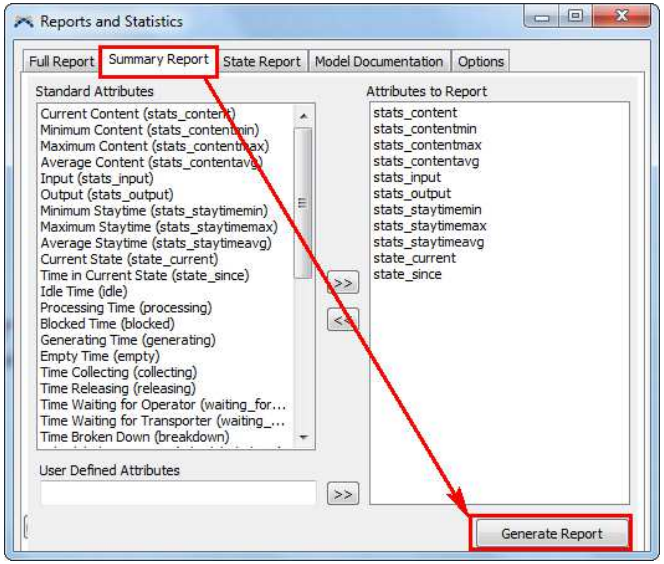
匯出數據統計表格 I

- 工具列 Statistics → Reports and Statistics



- 選擇 Summary Report 然後按下 Generate Report

匯出數據統計表格 II



- summary report 匯出為 Excel file 黃色區域分別為 L_q and W_q

匯出數據統計表格 III

	A	B	F	K
1	Flexsim Summary Report			
2	Time:	100000		
3				
4	Object	Class	stats_contentavg	stats_staytimeavg
5	Source1	Source	1	0
6	Queue2	Queue	1.394125	42.059366
7	Processor3	Processor	0.667082	20.128169
8	Sink4	Sink	0	0
9				

M/M/1: Analytical Results

λ	μ	$E(W)$	$E(L)$	$E(W_q)$	$E(L_q)$
2	3	1	2	2/3 $\sim$ 0.67	4/3 $\sim$ 1.333
1	2	1.00	1.00	0.50	0.50
0.1	2	0.53	0.05	0.03	0.00
0.5	2	0.67	0.33	0.17	0.08
0.5	1	2.00	1.00	1.00	0.50
2	3	1.00	2.00	0.67	1.33

Figure : M/M/1 analytical solution with different λ and μ, unit: minutes

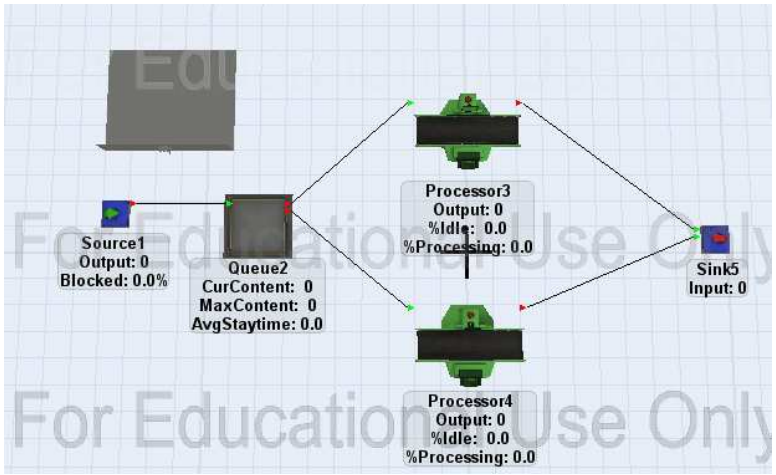
M/M/1: Analytical and Simulation

M/M/1: λ =2, $\mu = 3$

	Analytic Results	Simulation Results
$E(L)$	2	$1.394 + 0.667 = 2.061$
$E(W)$	1	$\frac{42.059+20.128}{60} \approx 1.036$
$E(L_q)$	$\frac{4}{3} \approx 1.333$	1.394
$E(W_q)$	$\frac{2}{3} \approx 0.667$	$\frac{42.059}{60} \approx 0.701$

Practice M/M/2 model

- Find the analytical solutions for M/M/2
- Create M/M/2 vis FlexSim



HomeWork

- Use FlexSim to generate estimates of $E(W)$, $E(L)$, $E(W_q)$, $E(L_q)$ from Models below.
- Due: 3/17.
- Models
  - M/M/2
  - G/G/1 (G: general distribution)
  - G/G/2
- Discussion. Set up the parameters you like. Comparing simulation results with analytical results, if possible.