

# CHAPTER 12



## LAPLACE TRANSFORM AND ITS APPLICATION IN CIRCUIT ANALYSIS

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## 12.1 Definition of the Laplace Transform

Pierre Simon Laplace (1749-1827) :

A French astronomer and mathematician *First* presented the Laplace transform and its applications to differential equations in 1779.

## 12.1 Definition of the Laplace Transform

Definition:

$$L[f(t)] = F(s) = \int_{0^-}^{\infty} f(t)e^{-st} dt$$

$s = \sigma + j\omega$  a complex variable

The Laplace transform is an integral transformation of a function  $f(t)$  from the time domain into the complex frequency domain,  $F(s)$ .

## 12.1 Definition of the Laplace Transform

One-sided (unilateral) Laplace transform

Two-sided (bilateral) Laplace transform

$$L^{-1}[F(s)] = f(t) = \frac{1}{2\pi j} \int_{s_1 - j\omega}^{s_1 + j\omega} F(s)e^{st} ds$$

Look-up table, an easier way for circuit application

$$f(t) \Leftrightarrow F(s)$$

## 12.1 Definition of the Laplace Transform

Similar to the application of phasor transform to solve the steady state AC circuits, Laplace transform can be used to transform the time domain circuits into S domain circuits to simplify the solution of integral differential equations to the manipulation of a set of algebraic equations.

## 12.2 Useful Laplace Transform Pairs

Functions	$f(t), t > 0^-$	F(s)
impulse	$\delta(t)$	1
step	$u(t)$	$\frac{1}{s}$
ramp	t	$\frac{1}{s^2}$
exponential	$e^{-at}$	$\frac{1}{s+a}$
sine	$\sin wt$	$\frac{w}{s^2 + w^2}$

## 12.2 Useful Laplace Transform Pairs

Functions	$f(t), t > 0^-$	$F(s)$
cosine	$\cos Wt$	$\frac{S}{S^2 + W^2}$
damped ramp	$te^{-at}$	$\frac{1}{(S+a)^2}$
damped sine	$e^{-at} \sin Wt$	$\frac{W}{(S+a)^2 + W^2}$
damped cosine	$e^{-at} \cos Wt$	$\frac{S+a}{(S+a)^2 + W^2}$

## 12.2 Useful Laplace Transform Pairs

$$\Delta L \left[ \frac{d}{dt} f(t) \right] = sF(s) - f(0^-)$$

$$\Delta L \left[ \int_{0^-}^t f(t) dt \right] = \frac{F(s)}{S}$$

$$\Delta L [f(t-a)u(t-a)] = e^{-as} F(s), a > 0$$

$$\Delta L [e^{-at} f(t)] = F(s+a)$$

$$\Delta L [f(at)] = \frac{1}{a} F\left(\frac{s}{a}\right), a > 0$$

$$\Delta \lim_{t \rightarrow 0^+} [f(t)] = \lim_{s \rightarrow \infty} sF(s)$$

$$\Delta \lim_{t \rightarrow \infty} [f(t)] = \lim_{s \rightarrow 0} sF(s)$$

## 12.2 Useful Laplace Transform Pairs

Example Use the Laplace transform to solve the differential equation.

$$\frac{d^2v}{dt^2} + 6\frac{dv}{dt} + 8v = 2u(t)$$

$$v(0) = 1$$

$$v'(0) = -2$$

Take Laplace transform

$$\left[ s^2V(s) - sv(0) - v'(0) \right] + 6[sV(s) - v(0)] + 8V(s) = \frac{2}{s}$$

## 12.2 Useful Laplace Transform Pairs

$$\left[ s^2V(s) - sv(0) - v'(0) \right] + 6[sV(s) - v(0)] + 8V(s) = \frac{2}{s}$$

$$(s^2 + 6s + 8)V(s) = \frac{s^2 + 4s + 2}{s}$$

$$\therefore V(s) = \frac{s^2 + 4s + 2}{s(s^2 + 6s + 8)} = \frac{s^2 + 4s + 2}{s(s+2)(s+4)}$$

$$v(t) = \frac{1}{4}(1 + 2e^{-2t} + e^{-4t})u(t)$$

## 12.3 Circuit Analysis in S Domain

(1) KCL ,  $\sum_n i_k(t) = 0$  , for any node.

Take Laplace Transform

$$\sum_n I_k(s) = 0 , \text{ for any node.}$$

(2) KVL ,  $\sum_m v_k(t) = 0$  , for any loop.

Take Laplace Transform

$$\sum_m V_k(s) = 0 , \text{ for any loop.}$$

## 12.3 Circuit Analysis in S Domain

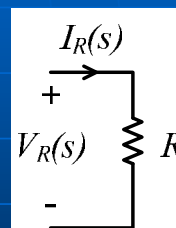
(3) Circuit Component Models

resistor

$$v_R(t) = Ri_R(t)$$

$$V_R(s) = RI_R(s)$$

$$I_R(s) = GV_R(s)$$



## 12.3 Circuit Analysis in S Domain

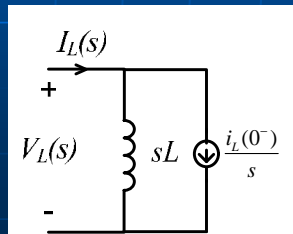
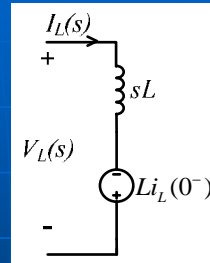
inductor

$$v_L(t) = L \frac{di_L}{dt}$$

$$i_L(t) = i_L(0^-) + \frac{1}{L} \int_{0^-}^t v_L(t) dt$$

$$V_L(s) = sLI_L(s) - Li_L(0^-)$$

$$I_L(s) = \frac{V_L(s)}{sL} + \frac{i_L(0^-)}{s}$$



## 12.3 Circuit Analysis in S Domain

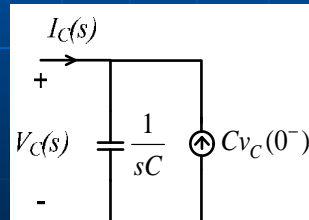
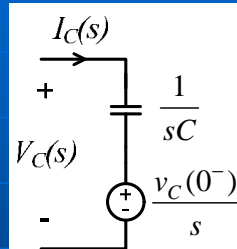
capacitor

$$i_C = C \frac{dv_C}{dt}$$

$$v_C(t) = v_C(0^-) + \frac{1}{C} \int_{0^-}^t i_C(t) dt$$

$$I_C(s) = sCV_C(s) - Cv_C(0^-)$$

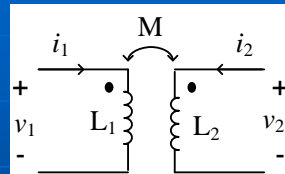
$$V_C(s) = \frac{v_C(0^-)}{s} + \frac{1}{sC} I_C(s)$$





## 12.3 Circuit Analysis in S Domain

Coupling inductors

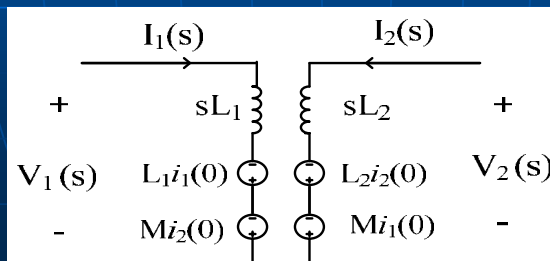


$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

$$V_1(s) = L_1 s I_1(s) - L_1 i_1(0) + M s I_2(s) - M i_2(0)$$

$$V_2(s) = M s I_1(s) - M i_1(0) + L_2 s I_2(s) - L_2 i_2(0)$$

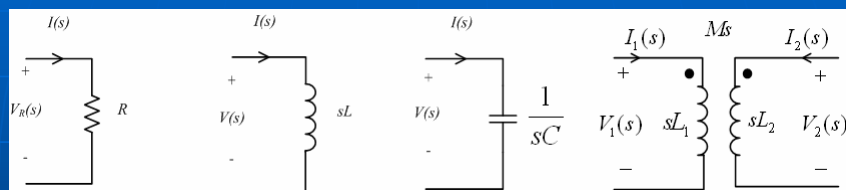


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## 12.3 Circuit Analysis in S Domain

For zero initial conditions



impedance @  $\frac{V(s)}{I(s)} = Z(s)$

admittance @  $\frac{I(s)}{V(s)} = Y(s) = \frac{1}{Z(s)}$

$V(s) = Z(s)I(s)$  ohm's law in s-domain

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## 12.3 Circuit Analysis in S Domain

The elegance of using the Laplace transform in circuit analysis lies in the automatic inclusion of the initial conditions in the transformation process, thus providing a complete (transient and steady state) solution.

## 12.3 Circuit Analysis in S Domain

Circuit analysis in s domain

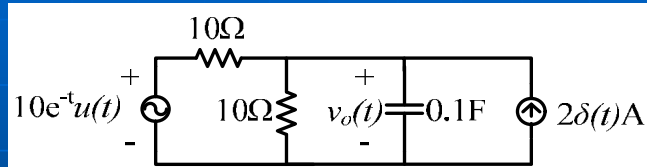
nStep 1 : Transform the time domain circuit into s-domain circuit.

nStep 2 : Solve the s-domain circuit.  
e.g. Nodal analysis or mesh analysis.

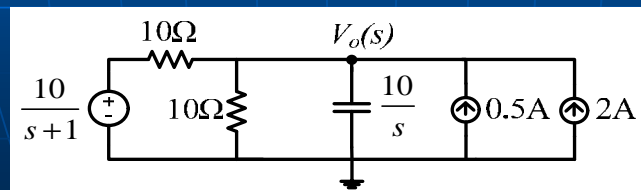
nStep 3 : Transform the solution back into time domain.

## 12.3 Circuit Analysis in S Domain

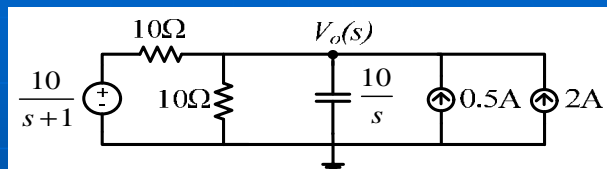
Example Find  $v_o(t)$  given  $v_o(0)=5V$



S-domain equivalent circuit



## 12.3 Circuit Analysis in S Domain



Nodal analysis

$$\frac{10}{s+1} - \frac{V_o(s)}{10} + 2 + 0.5 = \frac{V_o(s)}{10} + \frac{V_o(s)}{10/s}$$

$$\therefore V_o(s) = \frac{25s+35}{(s+1)(s+2)} = \frac{10}{s+1} + \frac{15}{s+2}$$

$$\therefore v_o(t) = (10e^{-t} + 15e^{-2t})u(t) \text{ V}$$

## 12.4 The Transfer Function and the Convolution Integral

Given a linear circuit N in s domain as shown below



Transfer function H(s) is defined as

$$H(s) = \frac{Y(s)}{X(s)} \Big|_{\text{With zero initial condition}}$$

## 12.4 The Transfer Function and the Convolution Integral

*If  $Y(s) = V_o(s)$  ,  $X(s) = V_i(s)$  ; then  $H(s) = \text{voltage gain}$*

*If  $Y(s) = I_o(s)$  ,  $X(s) = I_i(s)$  ; then  $H(s) = \text{current gain}$*

*If  $Y(s) = V(s)$  ,  $X(s) = I(s)$  ; then  $H(s) = \text{impedance}$*

*If  $Y(s) = I(s)$  ,  $X(s) = V(s)$  ; then  $H(s) = \text{admittance}$*

## 12.4 The Transfer Function and the Convolution Integral

Given the transfer function  $H(s)$  and input  $X(s)$ , then  $Y(s)=H(s)X(s)$

If the input is  $\delta(t)$ , then  $X(s)=1$  and  $Y(s)=H(s)$

Hence, the physical meaning of  $H(s)$  is in fact the Laplace transform of the impulse response of the corresponding circuit.

## 12.4 The Transfer Function and the Convolution Integral

$Y(s)=H(s)X(s)$ , in s-domain

$$y(t) = \int_{-\infty}^{\infty} h(t-t)x(t)dt @ h(t) * x(t) \text{ in time domain}$$

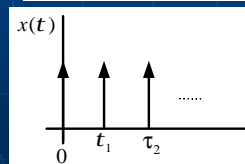
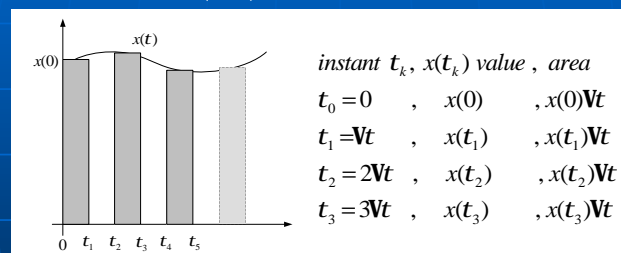
Geometrical interpretation of finding the convolution integral value at  $t=t_k$  is based on :

## 12.4 The Transfer Function and the Convolution Integral

- (1) Approximating the input function by using a series of impulse functions.
- (2) Shifting property of linear systems input  
 $x(t) \rightarrow \text{output } y(t)$   
 $x(t - \tau) \rightarrow \text{output } y(t - \tau)$
- (3) Superposition theorem for linear systems
- (4) Definition of integral : finding the area

## 12.4 The Transfer Function and the Convolution Integral

- (1) Input  $x(\tau)$  is approximated using impulse functions,  $x(\tau) = 0$ , for  $\tau < 0$



$$x(t) \cong f_0 d(t) + f_1 d(t - t_1) + f_2 d(t - t_2) + \dots$$

$$\text{@ } \sum_{k=0}^{\infty} x(k\Delta t) \Delta t d(t - k\Delta t)$$

## 12.4 The Transfer Function and the Convolution Integral

(2) Use the linearity property

<i>input</i>	→	<i>output(response)</i>
$x(0)\Delta t$	→	$x(0)\Delta t h(t)$
$x(t_1)\Delta t$	→	$x(t_1)\Delta t h(t - t_1)$
$x(t_2)\Delta t$	→	$x(t_2)\Delta t h(t - t_2)$
<b>M</b>		<b>M</b>
up to $t = t_k$		

## 12.4 The Transfer Function and the Convolution Integral

(3) Use superposition theorem to find the total approximate response

$$y(t_k) = \sum_{k=0}^n x(k\Delta t) \Delta t h(t_k - k\Delta t)$$

$$n = \text{integer}\left[\frac{t_k}{\Delta t}\right]$$

## 12.4 The Transfer Function and the Convolution Integral

(4) Take the limit,  $\Delta \tau \rightarrow d\tau$ ,  $\sum y(t_k) \longrightarrow y(t_k)$

$$y(t_k) = \int_{-\infty}^{\infty} x(t)h(t_k - t)dt$$

Due to causality principle,  $h(t - \tau) = 0$  for  $t \geq t_k$   
and  $x(\tau) = 0$  for  $\tau < 0$

$$= \int_0^{t_k} h(t_k - t)x(t)dt$$

## 12.4 The Transfer Function and the Convolution Integral

*Example:*

Given  $x(t) = u(t)$ ,  $h(t) = e^{-t}u(t)$ , find  $y(4)$ ,

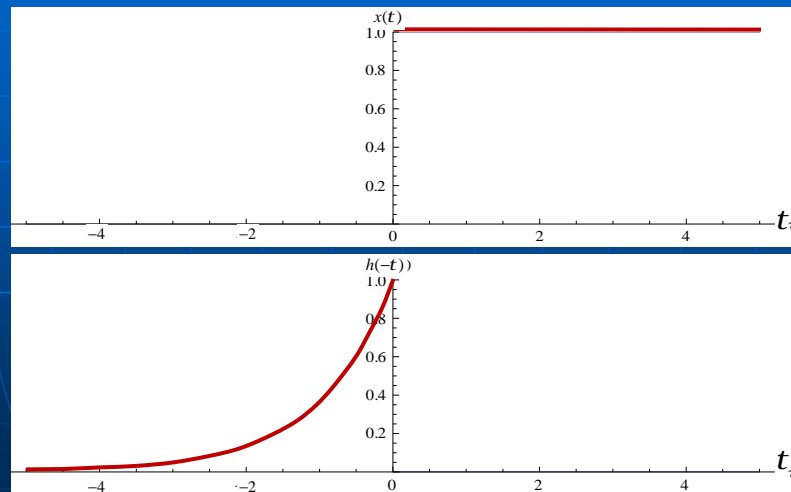
$$y(t) = \int_{-\infty}^{\infty} h(t-t)x(t)dt$$

$$\therefore y(4) = \int_{-\infty}^{\infty} h(4-t)x(t)dt$$



## 12.4 The Transfer Function and the Convolution Integral

Step 1.

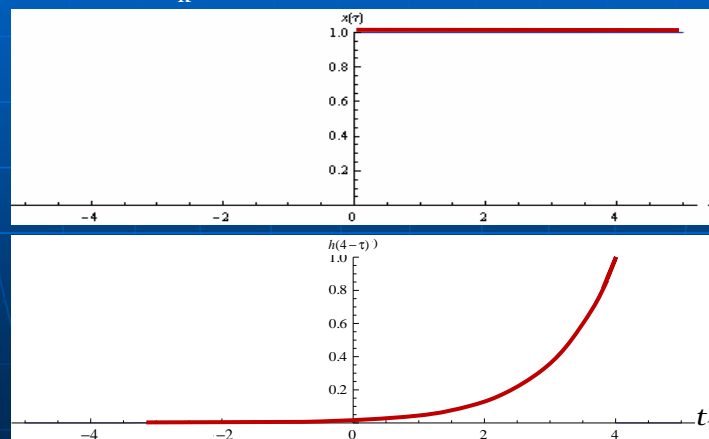


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## 12.4 The Transfer Function and the Convolution Integral

Step 2. Shift to  $t_k=4$

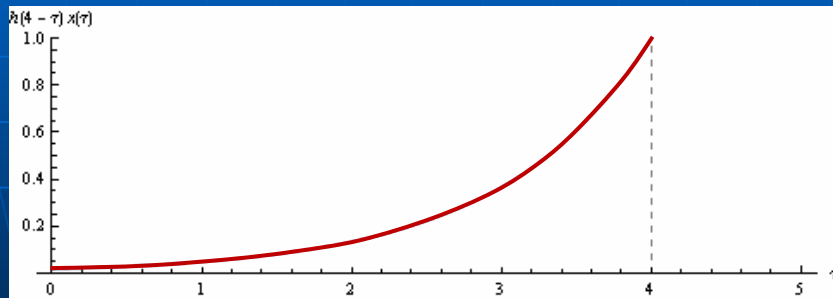


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## 12.4 The Transfer Function and the Convolution Integral

Step3. Find the product  $h(4-\tau)x(\tau)$



## 12.4 The Transfer Function and the Convolution Integral

Step4. Find the integral (area)

$$\begin{aligned} y(4) &= \int_0^4 e^{-(4-t)} dt = e^{-4} \int_0^4 e^t dt = e^{-4} e^t \Big|_0^4 \\ &= e^{-4} (e^4 - 1) = (1 - e^{-4}) \end{aligned}$$

Step5. Check

$$Y(s) = H(s)X(s) = \frac{1}{s+a} \times \frac{1}{s} = \frac{-1}{s+a} + \frac{1}{s}$$

$$\therefore y(t) = \frac{1}{a} (1 - e^{-t}), \quad a=1$$

$$\therefore y(4) = (1 - e^{-4})$$

## 12.5 The Transfer Function and the Steady State Sinusoidal Response

From definition of transfer function

$$H(s) = \frac{Y(s)}{X(s)}$$

$$\Rightarrow Y(s) = H(s)X(s)$$

Assume input  $X(t) = A \cos(\omega t + \Phi)$  and  $H(s)$  is given, then one can get the steady state solution without needing a separate phasor analysis.

## 12.5 The Transfer Function and the Steady State Sinusoidal Response

*proof* :  $X(t) = A \cos f \cos \omega t - A \sin f \sin \omega t$

$$\therefore X(s) = \frac{A(\cos f s - \sin f \omega)}{s^2 + \omega^2}$$

$$\therefore Y(s) = H(s)X(s)$$

$$= \frac{K_1}{s - j\omega} + \frac{K_1^*}{s + j\omega}$$

+  $\Sigma$  other terms due to poles

under steady state :

$$\therefore Y_{ss}(s) = \frac{K_1}{s - j\omega} + \frac{K_1^*}{s + j\omega}$$

$$K_1 = \left. \frac{H(s)A(s \cos f - \omega \sin f)}{s + j\omega} \right|_{s=j\omega}$$

$$= \frac{1}{2} H(j\omega) A e^{jf}$$

## 12.5 The Transfer Function and the Steady State Sinusoidal Response

Let  $H(j\omega) = |H(j\omega)| e^{jq(\omega)}$

and take inverse Laplace transform

Then  $y_{ss}(t) = A |H(j\omega)| \cos[\omega t + f + q(\omega)]$

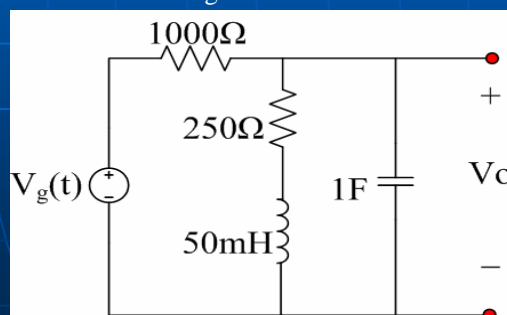
i.e.  $P(X(t)) = A \angle f$

then  $P(y_{ss}(t)) = A |H(j\omega)| \angle \underline{f + q(\omega)}$

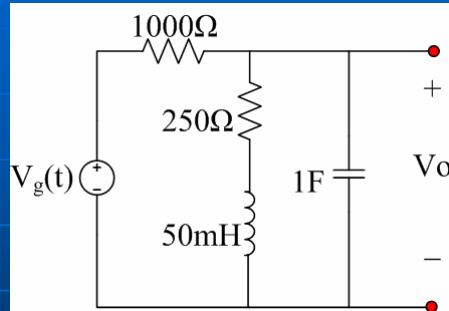
## 12.5 The Transfer Function and the Steady State Sinusoidal Response

Example : The transfer function  $H(s)$  of the circuit given below is known.

Find the steady state solution of  $V_o(t)$  for the given  $V_g(t)$ .



## 12.5 The Transfer Function and the Steady State Sinusoidal Response



$$H(s) = \frac{1000(s + 5000)}{s^2 + 6000s + 25 \cdot 10^6}$$

$$V_g(t) = 120 \cos(5000t + 30^\circ) \text{ V}$$

## 12.5 The Transfer Function and the Steady State Sinusoidal Response

*Solution:* Let  $s = j\omega = j5000$

$$\begin{aligned} \text{Evaluate } H(j5000) &= \frac{1000(j5000 + 5000)}{-25 \times 10^6 + j5000(6000) + 25 \times 10^6} \\ &= \frac{\sqrt{2}}{6} \angle -45^\circ \end{aligned}$$

$$\begin{aligned} \text{Then } V_o^{ss}(t) &= 120 \times \frac{\sqrt{2}}{6} \cos(5000t + 30^\circ - 45^\circ) \\ &= 20\sqrt{2} \cos(5000t - 15^\circ) \text{ V} \end{aligned}$$

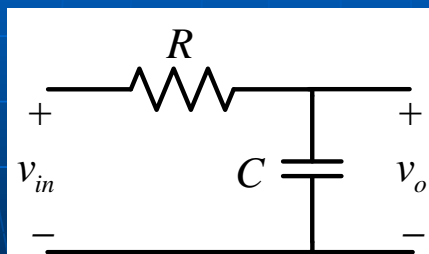
## 12.5 The Transfer Function and the Steady State Sinusoidal Response

In theory, the relationship between  $H(s)$  and  $H(j\omega)$  provides a link between the time domain and the frequency domain.

In some cases, we can determine  $H(j\omega)$  experimentally and then construct  $H(s)$  from the data.

## 12.5 The Transfer Function and the Steady State Sinusoidal Response

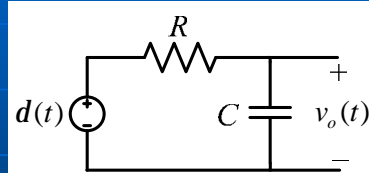
Example: Find the impulse response of the following circuit.



## 12.5 The Transfer Function and the Steady State Sinusoidal Response

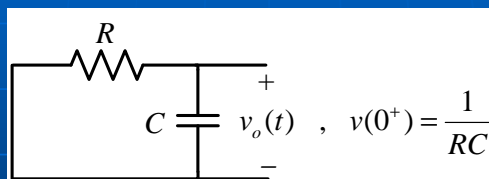
(a) Time domain solution

$$RC \frac{dv_o}{dt} + v_o = d(t)$$
$$\text{At } t = 0^-, v_o(0^-) = 0$$
$$\text{At } t = 0^+, v_o(0^+) = \frac{1}{C} \int_0^t \frac{d(t)}{R} dt$$
$$= \frac{1}{RC} \text{ V}$$



## 12.5 The Transfer Function and the Steady State Sinusoidal Response

For  $t > 0^+$ ,  $d(t) = 0$



$$RC \frac{dv_o}{dt} + v_o = 0$$
$$\therefore v_o(t) = \frac{1}{RC} e^{-\frac{t}{RC}} u(t)$$

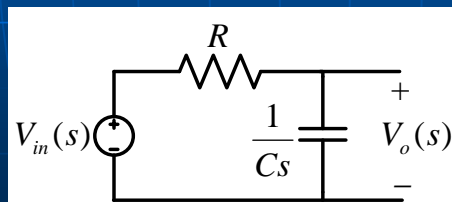
## 12.5 The Transfer Function and the Steady State Sinusoidal Response

(b) s-domain solution

Find the transfer function

$$H(s) = \left. \frac{V_o(s)}{V_{in}(s)} \right|_{\text{zero I.C.}}$$

Transform into s-domain circuit

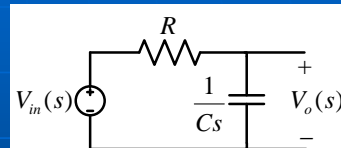


## 12.5 The Transfer Function and the Steady State Sinusoidal Response

$$\therefore H(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{\frac{1}{Cs}}{R + \frac{1}{Cs}} = \frac{1}{1 + RCs}$$

$$\begin{aligned} \therefore h(t) &= L^{-1}[H(s)] \\ &= \frac{1}{RC} e^{-\frac{t}{RC}} u(t) \end{aligned}$$

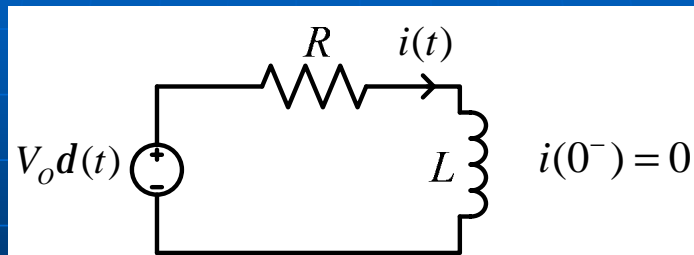
Same answer.





## 12.6 The Impulse Function in Circuit Analysis

Example 1: Impulse voltage source excitation

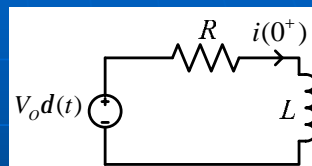


## 12.6 The Impulse Function in Circuit Analysis

(a) Time domain solution

$$\text{At } t = 0, i(0^-) = 0$$

$$\begin{aligned} \therefore i(0^+) &= \frac{1}{L} \int_{0^-}^t V_o d(x) dx \\ &= \frac{V_o}{L} \text{ (A)} \end{aligned}$$



The impulse voltage source has stored energy,  $\frac{1}{2} L (i(0^+))^2$ , in the inductor as an initial current in an infinitesimal moment.

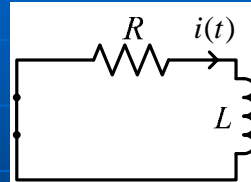
## 12.6 The Impulse Function in Circuit Analysis

For  $t > 0^+$ ,  $d(t) = 0$

$$L \frac{di}{dt} + Ri = 0, \text{ natural response}$$

$$i(0^+) = \frac{V_o}{L} \text{ A}$$

$$\therefore i(t) = \frac{V_o}{L} e^{-t/\tau} u(t), \quad \tau = \frac{L}{R}$$



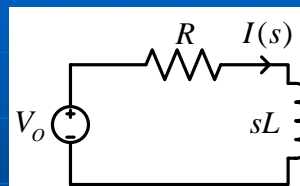
Note that the impulse source just builds up an initial inductor current but does not contribute to any forced response.

## 12.6 The Impulse Function in Circuit Analysis

(b) s-domain solution

$$\therefore I(s) = \frac{V_o}{R + sL} = \frac{V_o/L}{s + \frac{R}{L}}$$

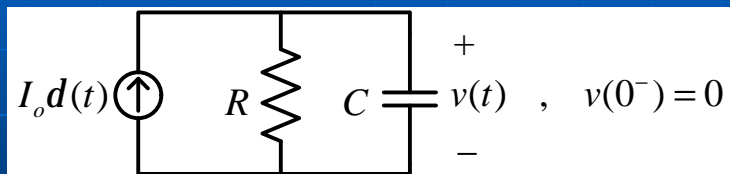
$$\therefore i(t) = \frac{V_o}{L} e^{-t/\tau} u(t)$$



Same answer but much easier.

## 12.6 The Impulse Function in Circuit Analysis

Example 2: Impulse current source excitation

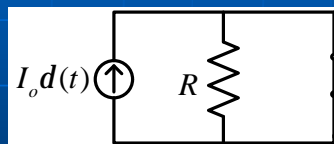


## 12.6 The Impulse Function in Circuit Analysis

(a) Time domain solution

$$\text{At } t = 0, v(0^-) = 0$$

$$\therefore v(0) = 0, \text{ short circuit}$$



$$\begin{aligned} v(0^+) &= \frac{1}{C} \int_0^t I_o d(x) dx \\ &= \frac{I_o}{C} \end{aligned}$$

The impulse current source has stored energy,  $\frac{1}{2} C (v(0^+))^2$ , in the capacitor as an initial voltage in an infinitesimal moment.

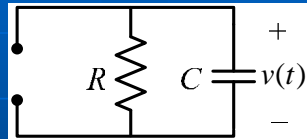
## 12.6 The Impulse Function in Circuit Analysis

For  $t > 0^+$ ,  $d(t) = 0$ , open circuit

$$C \frac{dv}{dt} + \frac{v}{R} = 0, \text{ natural response}$$

$$v(0^+) = \frac{I_o}{C}$$

$$\therefore v(t) = \frac{I_o}{C} e^{-\frac{t}{RC}} u(t), \quad t = RC$$

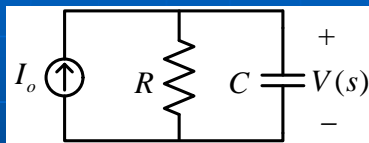


Note that the impulse current just builds up an initial capacitor voltage but does not contribute to any forced response.

## 12.6 The Impulse Function in Circuit Analysis

(b) s-domain solution

Transform into s-domain circuit



$$\therefore V(s) = I_o \frac{R \times \frac{1}{Cs}}{R + \frac{1}{Cs}} = \frac{I_o / C}{s + \frac{1}{RC}}$$

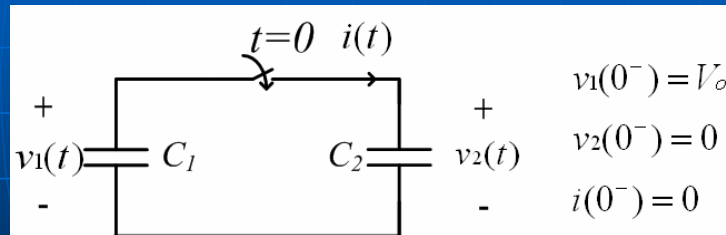
$$\therefore v(t) = \frac{I_o}{C} e^{-\frac{t}{RC}} u(t)$$

Same answer but much easier.

## 12.6 The Impulse Function in Circuit Analysis

Example 3: Impulse caused by switching operation

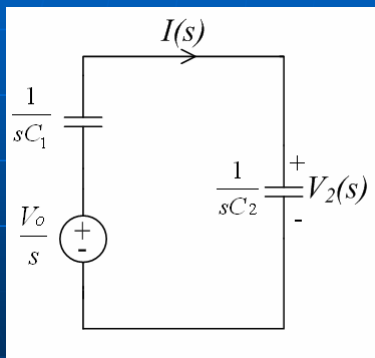
The switch is closed at  $t=0$  in the following circuit.



Note that  $v_1(0^-) \neq v_2(0^-)$

## 12.6 The Impulse Function in Circuit Analysis

Transform into s-domain



$$I(s) = \frac{V_o}{\frac{1}{sC_1} + \frac{1}{sC_2}} @ V_o \cdot C_e$$

$$C_e = \frac{C_1 \cdot C_2}{C_1 + C_2}, V_2(s) = \frac{V_o \cdot C_e}{sC_2} = \frac{V_o}{s} \cdot \frac{C_1}{C_1 + C_2}$$

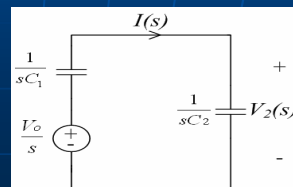
$$\therefore i(t) = V_o C_e d(t)$$

$$v_2(t) = \frac{C_1}{C_1 + C_2} V_o$$

## 12.6 The Impulse Function in Circuit Analysis

At  $t=0$ , a finite charge of  $C_1$  is transferred to  $C_2$  instantaneously.

Note that, as the switch is closed, the voltage across  $C_2$  does not jump to  $V_o$  of  $C_1$  but to its final value of the two paralleled capacitors.



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## 12.6 The Impulse Function in Circuit Analysis

$$\text{Note } Q_2 = C_2 \cdot V_2 = \frac{C_1 \cdot C_2}{C_1 + C_2} V_o, t > 0^+$$

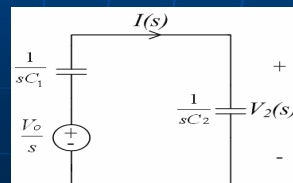
$$Q_1 = C_1 \cdot V_2 = \frac{C_1^2}{C_1 + C_2} V_o, t > 0^+$$

$$Q_1 + Q_2 = C_1 V_o, t > 0^+$$

Also, at  $t = 0^-$ ,  $Q_1 = C_1 V_o$ ,  $Q_2 = 0$

$$\therefore Q_1 + Q_2 = C_1 V_o$$

Conservation of charge.



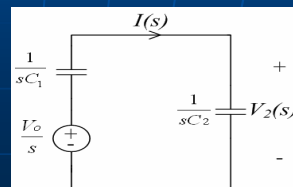
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## 12.6 The Impulse Function in Circuit Analysis

If we consider charged capacitors as voltage sources, then we should not connect two capacitors with unequal voltages in parallel.

Due to violation of KVL, an impulse will occur which may damage the components.



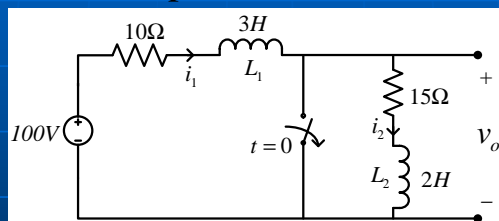
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## 12.6 The Impulse Function in Circuit Analysis

Example 4: Impulse caused by switching operation

The switch is opened at  $t = 0$  in the following circuit.



At  $t = 0^-$ , steady state solution

$$i_1(0^-) = \frac{100V}{10\Omega} = 10 A$$

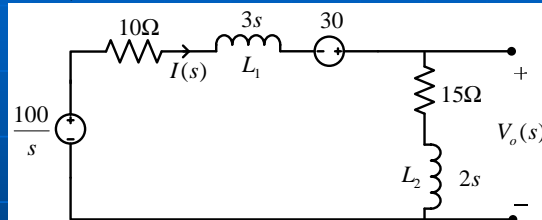
$$i_2(0^-) = 0 A$$

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## 12.6 The Impulse Function in Circuit Analysis

For  $t > 0$ , the S-domain circuit is



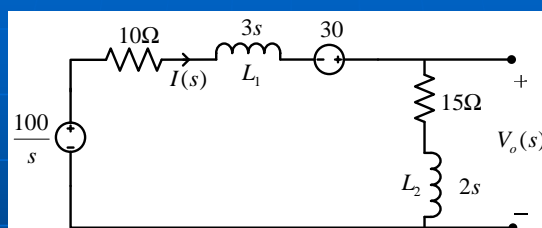
$$I(s) = \frac{(100/s) + 30}{5s + 25} = \frac{4}{s} + \frac{2}{s + 5}$$

$$V_o(s) = I(s) (2s + 15) = 12 + \frac{60}{s} + \frac{10}{s + 5}$$

$$\therefore i(t) = (4 + 2e^{-5t}) u(t) \dots\dots\dots(A)$$

$$v(t) = 12d(t) + (60 + 10e^{-5t}) u(t) \dots\dots(B)$$

## 12.6 The Impulse Function in Circuit Analysis



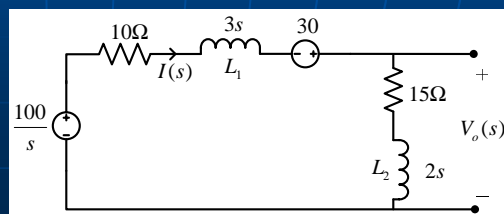
Note  $t = 0^-$ ,  $i_{L1}(0^-) = 10 \text{ A}$ ,  $i_{L2}(0^-) = 0 \text{ A}$   
 $t = 0^+$ , from (A),  $i_{L1}(0^+) = 6 \text{ A}$ ,  $i_{L2}(0^+) = 6 \text{ A}$   
 Also, from (B), there exists  $12d(t)$  at  $v_o(t)$ .



## 12.6 The Impulse Function in Circuit Analysis

Thus, if we consider an inductor current as a current source, then two inductors with unequal currents should not be connected in series.

Due to violation of KCL, it will result in impulse voltage which may damage the components.



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## SUMMARY

Objective 1 : Know the component models in s-domain.

Objective 2 : Be able to transform a time domain circuit into the s-domain circuit.

Objective 3 : Know how to analyze the s-domain circuit and transform the solution back to time domain.

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## SUMMARY

Objective 4 : Understand the significance of transfer function and be able to calculate the transfer function from the s-domain circuit.

Objective 5 : Know the geometrical interpretation of convolution integral and be able to calculate the integral.

## SUMMARY

Objective 6 : Know the relation between the phasor solution technique for finding sinusoidal steady state solution and the s-domain solution technique .

Objective 7 : Know how to use s-domain solution technique to solve a circuit containing impulse sources or a switching circuit which may result in impulse functions.

# SUMMARY

Chapter problems :

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13.27

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