CHAPTER 5

TWO-PORT CIRCUITS

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5.1 Definition of Two-Port Circuits

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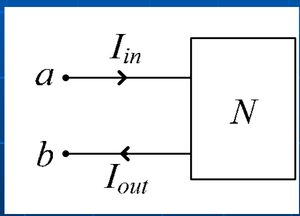
5.1 Definition of Two-Port Circuits

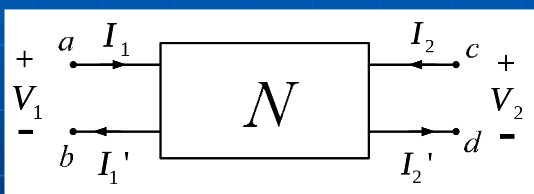
Consider a linear two-terminal circuit N consisting of no independent sources as follows :



For a, b two terminals, if $I_{in} = I_{out}$, then it constitutes a port.

5.1 Definition of Two-Port Circuits

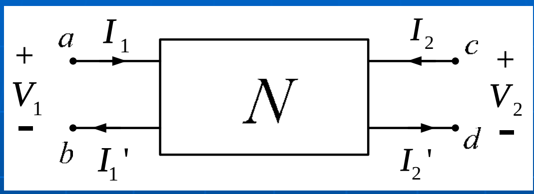
Now consider the following linear four-terminal circuit containing no independent sources.



with $I_1 = I_1'$
$I_2 = I_2'$

Then terminals a , b constitute the input port and terminals c , d constitute the output port.

5.1 Definition of Two-Port Circuits



No external connections exist between the input and output ports.

The two-port model is used to describe the performance of a circuit in terms of the voltage and current at its input and output ports.

5.1 Definition of Two-Port Circuits

Two-port circuits are useful in communications, control systems, power systems, and electronic systems.

They are also useful for facilitating cascaded design of more complex systems.

5.2 Classification of Two-Port Parameters

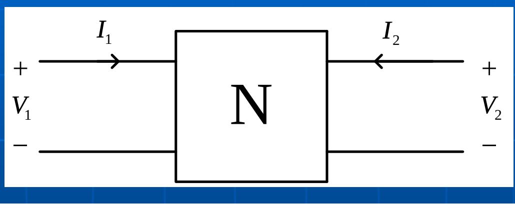
There are four terminal variables , namely V_1 ,V_2 , I_1 , I_2 , only two of them are independent.

Hence , there are only six possible sets of two-port parameters.

$$ {}_4C_2 = \frac{4\times 3}{2\times 1} = 6 $$

5.2 Classification of Two-Port Parameters

(1) The impedance , or Z , parameters

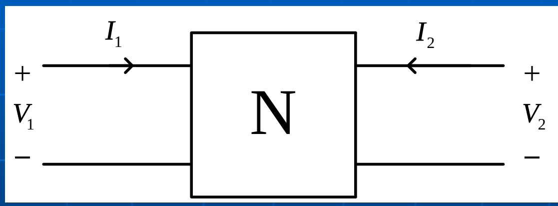


$$ \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \;,\; Z_{ij} : \; in \; \Omega $$

For two-port networks , four parameters are generally required to represent the circuit.

5.2 Classification of Two-Port Parameters

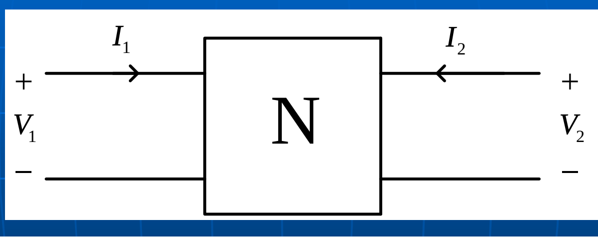
(2) The admittance , or Y , parameters



$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \; , \; Y_{ij} : in \; S$$

5.2 Classification of Two-Port Parameters

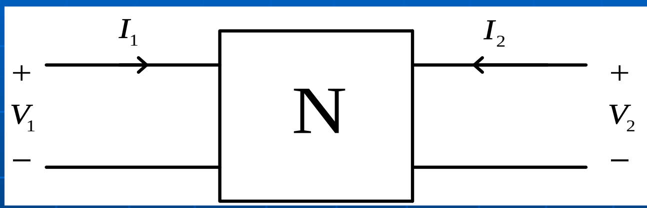
(3) The hybrid , or h , parameters



$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \; , \quad \begin{array}{l} h_{11} : in \; \Omega \\ h_{22} : in \; S \\ h_{12} \,\&\, h_{22} \; scalars \end{array}$$

5.2 Classification of Two-Port Parameters

(4) The inverse hybrid , or g , parameters



$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix},$$

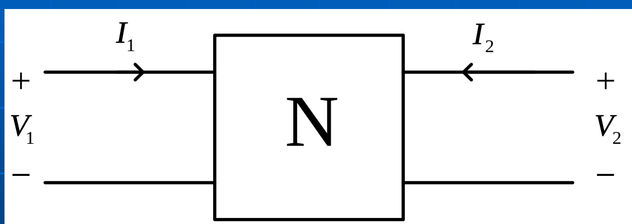
g_{11} : *in S*

g_{22} : *in* Ω

g_{12} & g_{21} *scalars*

5.2 Classification of Two-Port Parameters

(5) The transmission , or a , parameters



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix},$$

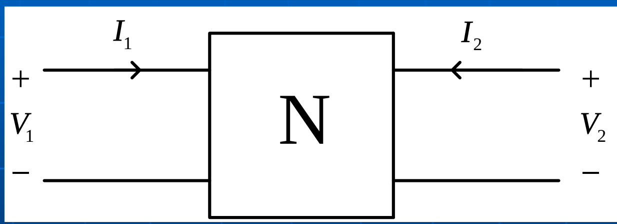
a_{12} : *in* Ω

a_{21} : *in S*

a_{11} & a_{22} *scalars*

5.2 Classification of Two-Port Parameters

(6) The inverse transmission , or b , parameters



$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}, \quad \begin{matrix} b_{12} \ : \ in \ \Omega \\ b_{21} \ : \ in \ S \\ b_{11} \ \& \ b_{22} \ scalars \end{matrix}$$

5.3 Finding Two-Port Parameters

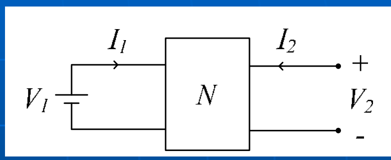
Method 1 : Calculate or measure by invoking appropriate short-circuit and open-circuit conditions at the input and output ports.

Method 2 : Derive the parameters from another set of two-port parameters.

5.3 Finding Two-Port Parameters

Method 1 : Choose Z parameters as an illustration.

$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$



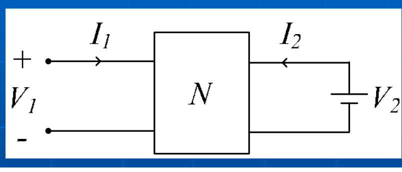
when $I_2 = 0$, output port is open

$$V_1 = z_{11}I_1 \text{ , } \backslash \; z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \text{ ,}\textit{input impedance}$$

$$V_2 = z_{21}I_1 \text{ , } \backslash \; z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \text{ ,}\textit{transfer impedance}$$

5.3 Finding Two-Port Parameters

When $I_1 = 0$, input port is open



$$V_1 = z_{12}I_2 \text{ , } \backslash \; z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \text{ ,}\textit{transfer impedance}$$

$$V_2 = z_{22}I_2 \text{ , } \backslash \; z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \text{ ,}\textit{output impedance}$$

When the two-port does not contain any dependent source, then $z_{12}=z_{21}$.

5.3 Finding Two-Port Parameters

z_{11} & z_{22} are called driving-point impedances.

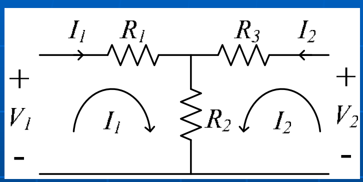
z_{12} & z_{21} are called transfer impedances.

When $z_{11}=z_{22}$, the two-port circuit is said to be symmetrical.

When $z_{12}=z_{21}$, the two-port circuit is called a reciprocal circuit.

5.3 Finding Two-Port Parameters

Example 1 : Find the Z parameters of the T-network



Assign mesh currents as shown :

$$\begin{bmatrix} R_1 + R_2 & R_2 \\ R_2 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

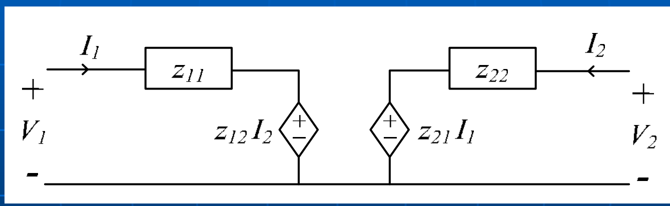
$$\therefore\ z_{11} = R_1 + R_2$$

$$z_{12} = z_{21} = R_2$$

$$z_{22} = R_2 + R_3$$

5.3 Finding Two-Port Parameters

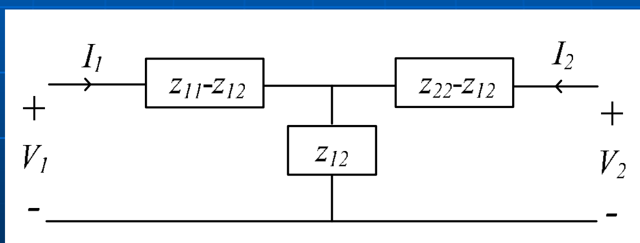
A two-port can be replaced by the following equivalent circuit.



$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

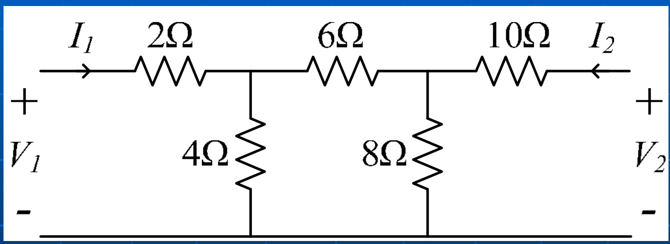
5.3 Finding Two-Port Parameters

In case the two-port is reciprocal, z_{12}=z_{21}, then it can also be represented by the T-equivalent circuit.



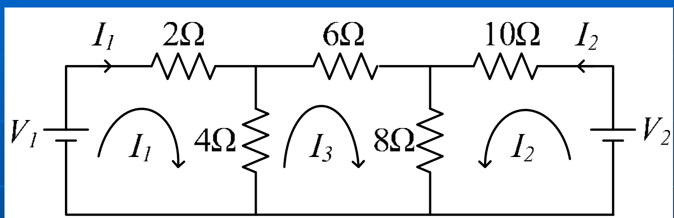
5.3 Finding Two-Port Parameters

Example 2 : Find the Z parameters



Assign mesh currents as follows and write down the mesh equation.

5.3 Finding Two-Port Parameters



$$\begin{bmatrix} 2+4 & 0 & -4 \\ 0 & 10+8 & 8 \\ -4 & 8 & 4+6+8 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \left[\begin{array}{c} I_1 \\ I_2 \\ \hline I_3 \end{array}\right] = \left[\begin{array}{c} V_1 \\ V_2 \\ \hline 0 \end{array}\right]$$

5.3 Finding Two-Port Parameters

$$A\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} + BI_3 = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad \text{...... (1)}$$

$$C\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} + DI_3 = 0 \quad \text{.......... (2)}$$

$$From\ (2)\ ,\quad I_3 = -D^{-1}C\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad \text{...... (3)}$$

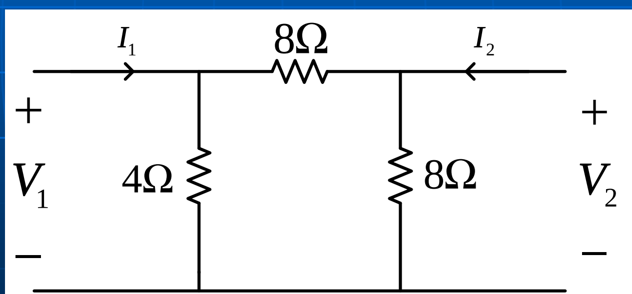
Substitute (3) *into* (1)

$$A\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} - BD^{-1}C\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} A - BD^{-1}C \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\therefore\ [Z] = A - BD^{-1}C$$

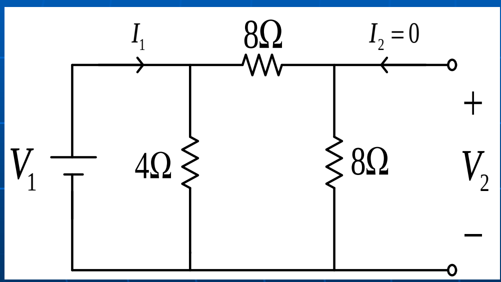
5.3 Finding Two-Port Parameters

Example 3: Find the Z parameters of the following circuit by definition of Z parameters.



5.3 Finding Two-Port Parameters

Step1: Let $I_2 = 0$ and apply V_1



$$I_1 = V_1 / (4\Omega // (8+8)\Omega) = \frac{20}{64} V_1$$

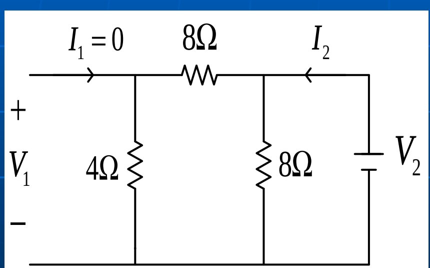
$$V_2 = \frac{8}{8+8} V_1 = \frac{1}{2} V_1$$

$$\therefore z_{11} = \frac{V_1}{I_1} = \frac{64}{20} = \frac{16}{5}\Omega$$

$$z_{21} = \frac{V_2}{I_1} = \frac{\frac{1}{2}V_1}{I_1} = \frac{1}{2} z_{11} = \frac{8}{5}\Omega$$

5.3 Finding Two-Port Parameters

Step2: Let $I_1 = 0$ and apply V_2



$$I_2 = V_2 / (8\Omega // (8+4)\Omega) = \frac{5}{24} V_2$$

$$V_1 = \frac{4}{4+8} V_2 = \frac{1}{3} V_2$$

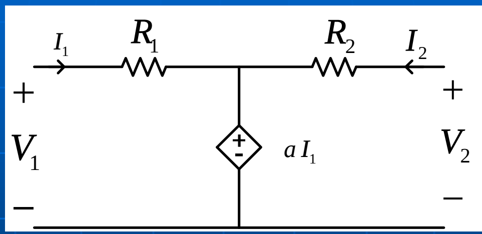
$$\therefore z_{22} = \frac{V_2}{I_2} = \frac{24}{5}\Omega$$

$$z_{12} = \frac{V_1}{I_2} = \frac{\frac{1}{3}V_2}{I_2} = \frac{1}{3} z_{22} = \frac{8}{5}\Omega$$

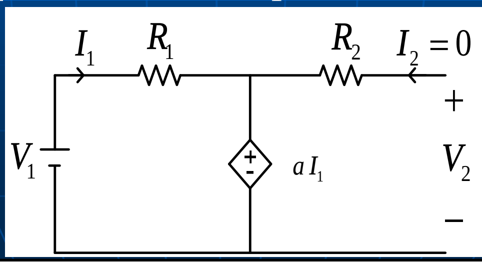
$$\therefore [Z] = \begin{bmatrix} 16/5 & 8/5 \\ 8/5 & 24/5 \end{bmatrix}\Omega$$

5.3 Finding Two-Port Parameters

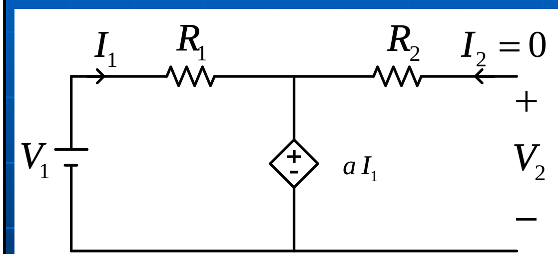
Example 4: Containing dependent source case



Step1: Let $I_2 = 0$ and apply V_1



5.3 Finding Two-Port Parameters



$$I_1 = \frac{V_1 - a I_1}{R_1}$$

$$\Rightarrow I_1 = \frac{V_1}{R_1 + a}$$

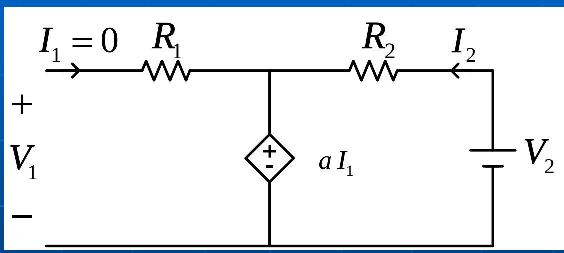
$$V_2 = a I_1$$

$$\therefore \; z_{11} = \frac{V_1}{I_1} = R_1 + a$$

$$z_{21} = \frac{V_2}{I_1} = a$$

5.3 Finding Two-Port Parameters

Step2: Let $I_1 = 0$ and apply V_2



$I_1 = 0$

$\therefore I_2 = \dfrac{V_2}{R_2}$

$V_1 = 0$

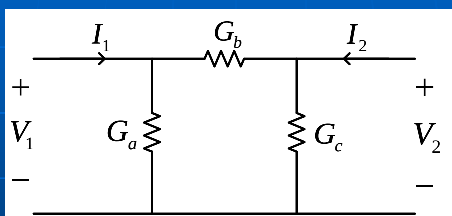
$$\therefore z_{12} = \frac{V_1}{I_2} = 0$$

$$z_{22} = \frac{V_2}{I_2} = R_2$$

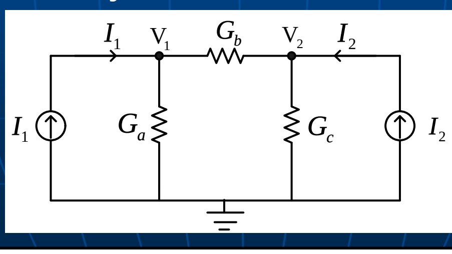
$$, \quad \therefore [Z] = \begin{bmatrix} R_1 + a & 0 \\ a & R_2 \end{bmatrix}$$

5.3 Finding Two-Port Parameters

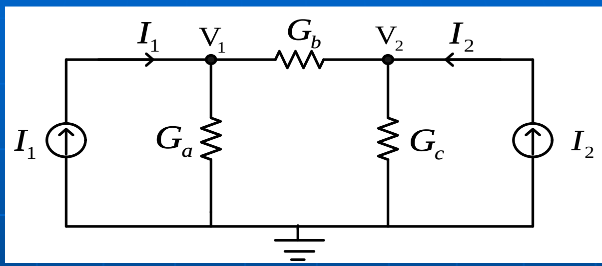
Example 5: Find the y parameters of the following circuit.



Use nodal analysis



5.3 Finding Two-Port Parameters



Nodal equation

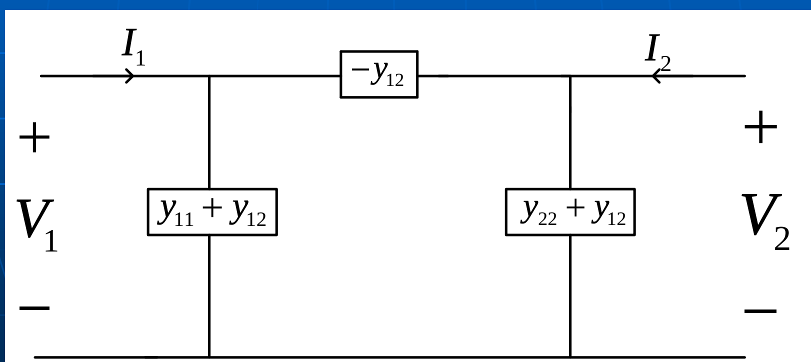
$$\begin{bmatrix} G_a + G_b & -G_b \\ -G_b & G_b + G_c \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\therefore [y] = \begin{bmatrix} G_a + G_b & -G_b \\ -G_b & G_b + G_c \end{bmatrix} \quad in \;\; S \;\; unit$$

$$Note\ that\ y_{12} = y_{21} = -G_b$$

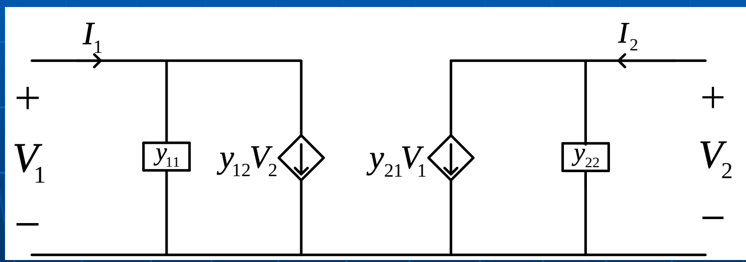
5.3 Finding Two-Port Parameters

A linear reciprocal two-port can be represented by the following equivalent ∏ circuit .



5.3 Finding Two-Port Parameters

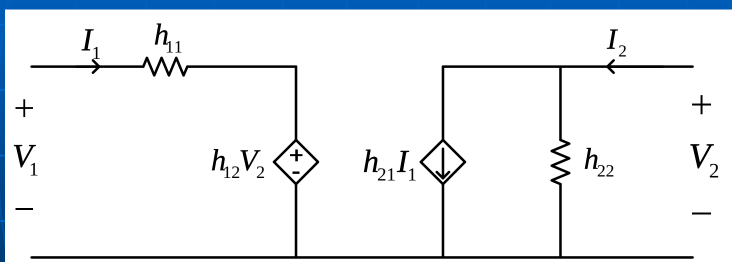
Similarly , a linear two-port can also be represented by the following equivalent circuit with dependent sources .



$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

5.3 Finding Two-Port Parameters

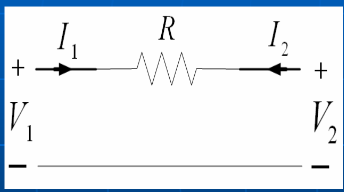
A linear two-port can be represented by the following equivalent circuit .



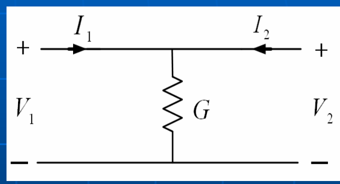
$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

5.3 Finding Two-Port Parameters

Example 6 : Find the transmission parameters of the following circuits



(a)

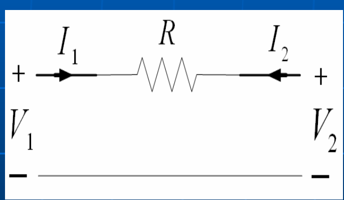


(b)

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

5.3 Finding Two-Port Parameters

For circuit (a) and when I_2=0

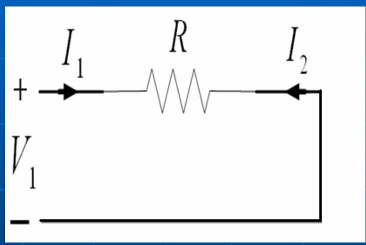


$$I_1 = -I_2 = 0 \quad \therefore a_{21} = \frac{I_1}{V_2} = 0$$

$$V_2 = V_1 \quad \therefore a_{11} = \frac{V_1}{V_2} = 1$$

5.3 Finding Two-Port Parameters

For circuit (a) and when $V_2=0$



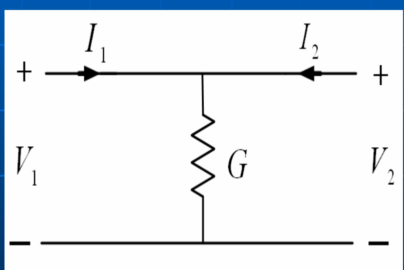
$$V_1 = RI_1 \quad \therefore a_{12} = \frac{V_1}{-I_2} = \frac{V_1}{I_1} = R$$

$$I_1 = -I_2 \quad \therefore a_{22} = \frac{I_1}{-I_2} = 1$$

$$\therefore \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix}$$

5.3 Finding Two-Port Parameters

Similarly, for circuit (b)



$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ G & 1 \end{bmatrix}$$

5.3 Finding Two-Port Parameters

Method (2)

The 6 sets of parameters relate the same input and output terminal variables, hence they are interrelated.

A systematical procedure for obtaining a set of parameters from another one is given as follows for reference.

5.3 Finding Two-Port Parameters

Step1: Arrange the given two port parameters in the following standard form:

$k_{11}V_1+k_{12}I_1+k_{13}V_2+k_{14}I_2=0$

$k_{21}V_1+k_{22}I_1+k_{23}V_2+k_{24}I_2=0$

Step2: Separate the independent variables and the dependent variables of the desired parameter set.

Step3: Find the solution of the dependent variable vector.

5.3 Finding Two-Port Parameters

Example 7: Given Z parameters, find h parameters.

Step1 From $V_1=z_{11}I_1+z_{12}I_2$

$V_2=z_{21}I_1+z_{22}I_2$

In the standard form

$1V_1-z_{11}I_1+0V_2-z_{12}I_2=0$

$0V_1+z_{21}I_1-1V_2+z_{22}I_2=0$

5.3 Finding Two-Port Parameters

Step2

$$\begin{bmatrix} 1 & -z_{12} \\ 0 & z_{22} \end{bmatrix}\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} z_{11} & 0 \\ -z_{21} & 1 \end{bmatrix}\begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

Step3 *solve* $\begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1 & -z_{12} \\ 0 & z_{22} \end{bmatrix}^{-1}\begin{bmatrix} z_{11} & 0 \\ -z_{21} & 1 \end{bmatrix}\begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$= \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

5.3 Finding Two-Port Parameters

A different solution approach

$$V_1 = z_{11}I_1 + z_{12}I_2 \quad ....(A)$$
$$V_2 = z_{21}I_1 + z_{22}I_2 \quad ....(B)$$

From (B) one can obtain

$$I_2 = \frac{-z_{21}}{z_{22}}I_1 + \frac{1}{z_{22}}V_2 \quad ....(C)$$
$$= h_{21}I_1 + h_{22}I_2$$

From (A) and (C)

$$V_1 = z_{11}I_1 + z_{12}(h_{21}I_1 + h_{22}I_2)$$
$$= (z_{11} + z_{12}h_{21})I_1 + z_{12}h_{22}I_2$$
$$@\ h_{11}I_1 + h_{12}V_2$$

5.3 Finding Two-Port Parameters

Example 8 :

Two sets of measurements are made on a two-port resistive circuit. The first set is made with port 2 open, and the second set is made with port 2 short-circuited. The results are as follows :

Port 2 open	Port 2 short-circuited
$V_1 = 10mV$	$V_1 = 24mV$
$I_1 = 10\mu A$	$I_1 = 20\mu A$
$V_2 = -40V$	$I_2 = 1mA$

Find h parameters from there measurements.

5.3 Finding Two-Port Parameters

$V_1 = h_{11} I_1 + h_{12} V_2$ .... (A)

$I_2 = h_{21} I_1 + h_{22} V_2$ .... (B)

When port 2 is short circuited, $V_2=0$

$V_1=24mV,\ I_1=20\mu A,\ I_2=1mA$

Hence, from (A) and (B)

$24mV = h_{11}(20\mu A) + 0$

$1mA = h_{21}(20\mu A) + 0$

$\therefore h_{11} = 1.2\ K\Omega\ ,\ h_{21} = 50$ ......(C)

5.3 Finding Two-Port Parameters

When port 2 is open, $I_2=0$

$V_1=10mV,\ I_1=10\mu A,\ V_2=-40V$

Hence, from (A), (B) and (C)

$10mV = 1.2\ K\Omega\ (10\mu A) + h_{12}\ (-40V)$

$0 = 50\ (10\mu A) + h_{22}\ (-40V)$

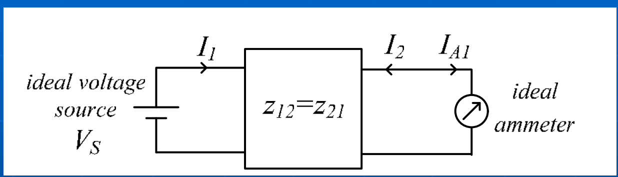
$\therefore h_{12} = 5x10^{-5}\ ,\ h_{22} = 12.5\ \mu S$

5.4 Analysis of the Terminated Two-Port Circuit

Reciprocal Theorem

Version 1 : For a reciprocal circuit, the interchange of an ideal voltage source at one port with an ideal ammeter at the other port produces the same ammeter reading.

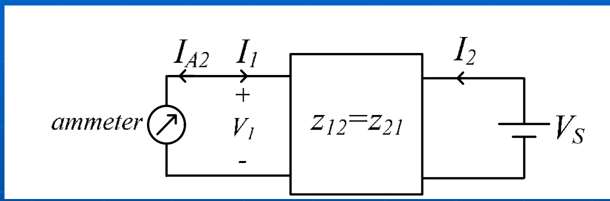
5.4 Analysis of the Terminated Two-Port Circuit



$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

$$\because \; V_1 = V_S \;,\; I_2 = -I_{A1} \;,\; V_2 = 0$$
$$V_S = z_{11}I_1 + z_{12}(-I_{A1})$$
$$0 = z_{12}I_1 + z_{22}(-I_{A1})$$
$$\therefore \; I_{A1} = \frac{z_{12}V_S}{z_{11}z_{22} - z_{12}^2}$$

5.4 Analysis of the Terminated Two-Port Circuit



$$\because\ V_1 = 0\ ,\ I_{A2} = -I_1\ \ ,\ V_2 = V_S$$

$$\therefore\ \ 0\ = z_{11}(-I_{A2}) + z_{12}I_2$$

$$V_S\ = z_{12}(-I_{A2}) + z_{22}I_2$$

$$\therefore\ I_{A2} = \frac{z_{12}V_S}{z_{11}z_{22} - z_{12}^2} = I_{A1}$$

5.4 Analysis of the Terminated Two-Port Circuit

The effect of reciprocity on the two-port parameters is given by

$$z_{12} = z_{21}\ \ ,\ \ or\ \ \ y_{12} = y_{21}$$

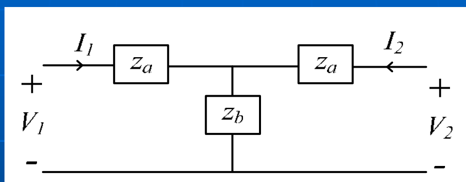
$$a_{11}a_{22} - a_{12}a_{21} = 1\ \ ,\ or$$

$$b_{11}b_{22} - b_{12}b_{21} = 1\ \ \ ,\ or$$

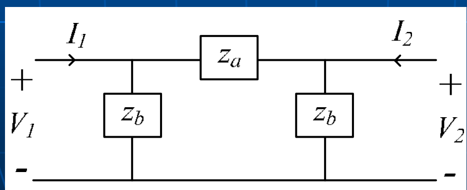
$$h_{12} = -h_{21}\ ,\ \ or\ \ \ g_{12} = -g_{21}$$

5.4 Analysis of the Terminated Two-Port Circuit

Examples of symmetric two ports.

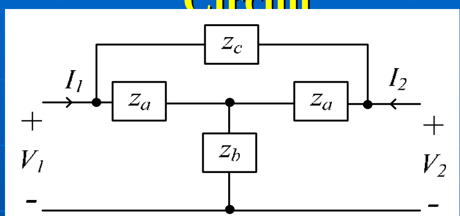


(a) symmetric tee

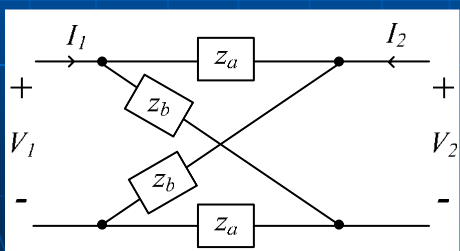


(b) symmetric pi

5.4 Analysis of the Terminated Two-Port Circuit



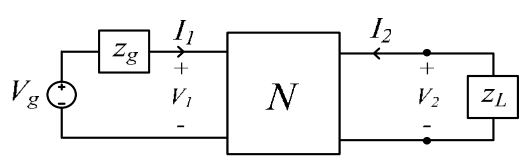
(c) symmetric bridged tee



(d) symmetric lattice

5.4 Analysis of the Terminated Two-Port Circuit

There are mainly 6 interested characteristics for a terminated two-port circuit in practical applications.



A terminated two-port model

5.4 Analysis of the Terminated Two-Port Circuit

input impedance Z_{in} @ V_1 / I_1

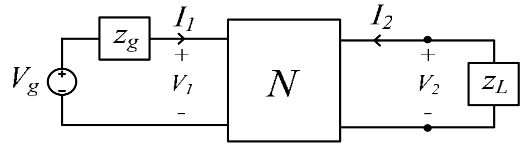
output current I_2

Thevenin equivalent looking into port 2.

current gain I_2 / I_1

voltage gain V_2 / V_1

voltage gain V_2 / V_g



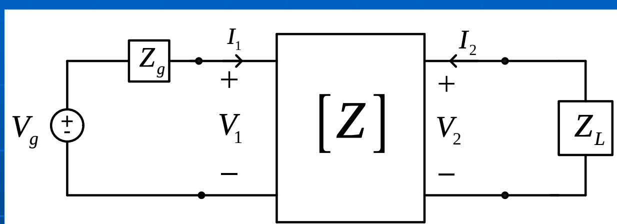
A terminated two-port model

5.4 Analysis of the Terminated Two-Port Circuit

The derivation of any one of the desired expressions involves the algebraic manipulation of the two-port equations along with the two constraint equations imposed at input and output terminals.

5.4 Analysis of the Terminated Two-Port Circuit

Example 9 : Use Z parameters as an illustration



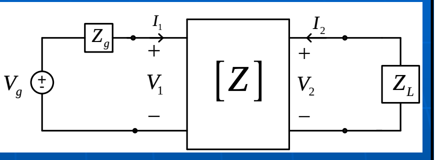
two-port equation

$$V_1 = z_{11}I_1 + z_{12}I_2 \quad \text{L L} \quad (A)$$
$$V_2 = z_{21}I_1 + z_{22}I_2 \quad \text{L L} \quad (B)$$

5.4 Analysis of the Terminated Two-Port Circuit

input port constraint

$$V_g = I_1 z_g + V_1 \quad \mathbf{L\,L} \quad (C)$$



Output port constraint

$$V_2 = -z_L I_2 \quad \mathbf{L\,L} \quad (D)$$

(1) Find $Z_{in} = V_1 / I_1$

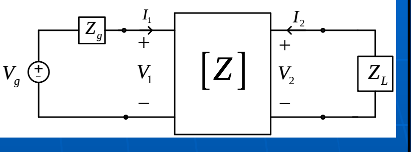
From (D) and (B) ,

$$I_2 = \frac{-z_{21} I_1}{z_{22} + z_L} \quad \mathbf{L\,L} \quad (E)$$

5.4 Analysis of the Terminated Two-Port Circuit

Substitute (E) into (A)

$$Z_{in} = z_{11} - \frac{z_{12} z_{21}}{z_{22} + z_L}$$



(2) Find I_2 : From (A) and (C)

$$I_1 = \frac{V_g - z_{12} I_2}{z_{11} + z_g} \quad \mathbf{L\,L} \quad (F)$$

Substitute (F) into (E)

$$I_2 = \frac{-z_{21} V_g}{(z_{11} + z_g)(z_{22} + z_L) - z_{12} z_{21}} \quad \mathbf{L\,L} \quad (G)$$

5.4 Analysis of the Terminated Two-Port Circuit

(3) Find V_{TH} and Z_{TH} at port 2 :

With I_2=0 , from (A) , (B) and (F)

$$V_1 = z_{11} I_1$$

$$V_2 = z_{21} I_1 \Rightarrow V_2 = \frac{z_{21}}{z_{11}} V_1 \quad \cdots\cdots \quad (H)$$

$$I_1 = \frac{V_g}{z_{11} + z_g} \cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (I)$$

From (C) , (H) and (I)

$$\therefore V_{TH} = V_2 = \frac{z_{21}}{z_g + z_{11}} V_g$$

5.4 Analysis of the Terminated Two-Port Circuit

With V_g =0 , then $V_1 = -z_g I_1 \quad \cdots\cdots \quad (J)$

From (A) and (J) , $I_1 = \dfrac{-z_{12} I_2}{z_{11} + z_g}$

$$\therefore Z_{TH} = \frac{V_2}{I_2} = z_{22} - \frac{z_{12} z_{21}}{z_{11} + z_g}$$

(4) Find current gain I_2/I_1

From (E) , $\dfrac{I_2}{I_1} = -\dfrac{z_{21}}{z_L + z_{22}}$

5.4 Analysis of the Terminated Two-Port Circuit

(5) Find voltage gain V_2/V_1 :

From (B) and (D)

$$V_2 = z_{21} I_1 + z_{22}\left(-\frac{V_2}{z_L}\right)$$

$$\therefore V_2 = \frac{z_{21} z_L}{z_L + z_{22}} I_1 \;\mathbf{L\,L\,L\,L\,L\,L\,L}\; \text{(K)}$$

From (A) , (D) , and (K)

$$V_1 = \frac{(z_{11} z_L + z_{11} z_{22} - z_{12}^2)}{z_L + z_{22}} I_1$$

$$\therefore \frac{V_2}{V_1} = \frac{z_{21} z_L}{z_{11} z_L + z_{11} z_{22} - z_{12}^2} \;\mathbf{L\,L\,L\,L\,L}\; (L)$$

5.4 Analysis of the Terminated Two-Port Circuit

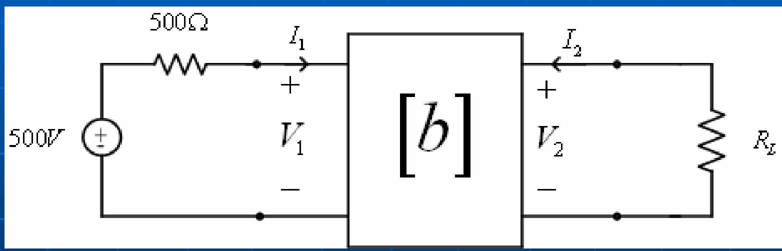
(6) Find V_2/V_g :

$$\mathbf{Q}\; \frac{V_2}{V_g} = \frac{V_2}{V_1}\frac{V_1}{V_g} = \frac{V_2}{V_1}\frac{Z_{in}}{(z_g + Z_{in})}$$

$$= \frac{z_{21} z_L}{(z_{11} + z_g)(z_{22} + z_L) - z_{12} z_{21}}$$

5.4 Analysis of the Terminated Two-Port Circuit

Example 10 : Given the following circuit, find V_2 when R_L=5KΩ

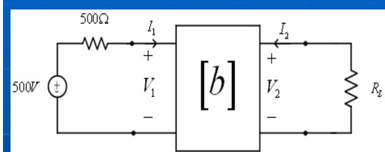


$$b_{11} = -20 \qquad b_{12} = -3000\Omega$$
$$b_{21} = -2ms \qquad b_{22} = -0.2$$

5.4 Analysis of the Terminated Two-Port Circuit

Example 10: (cont.)

$$V_2 = b_{11}V_1 - b_{12}I_1 \ldots\ldots\ldots\ldots(A)$$
$$I_2 = b_{21}V_1 - b_{22}I_1 \ldots\ldots\ldots\ldots(B)$$
$$V_1 = 500 - 500 I_1 \ldots\ldots\ldots\ldots(C)$$
$$I_2 = -\frac{V_2}{R_L} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(D)$$



Substitute (C) and (D) into (A) and (B) to eliminate V_1 and I_2

$$V_2 - 13\times10^3 I_1 = -10^4 \ldots..(E)$$
$$V_2 + 6\times10^3 I_1 = 5\times10^3 \ldots..(F)$$

From (E) and (F) , $V_2 = \dfrac{5000}{19} = 263.16V$

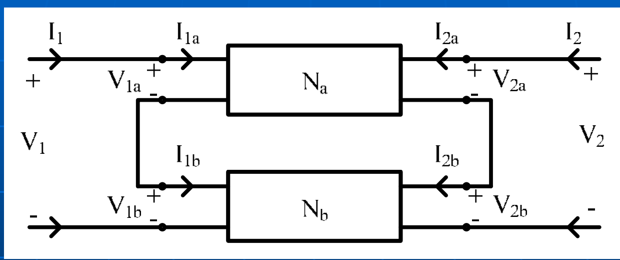
5.5 Interconnected Two-Port Circuits

Two-port circuits may be interconnected in five ways :

(1) in series
(2) in parallel
(3) in series-parallel
(4) in parallel-series
(5) in cascade

5.5 Interconnected Two-Port Circuits

(1) Series connection



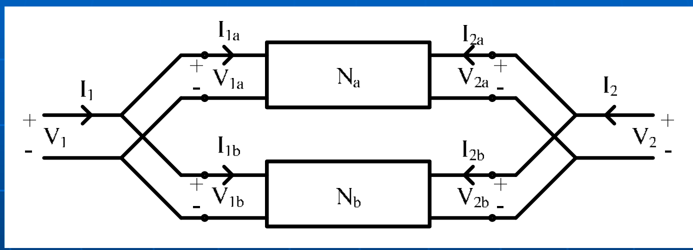
$$I_1 = I_{1a} = I_{1b} \qquad V_1 = V_{1a} + V_{1b}$$

$$I_2 = I_{2a} = I_{2b} \qquad V_2 = V_{2a} + V_{2b}$$

$$\therefore \quad [Z] = [Z_a] + [Z_b]$$

5.5 Interconnected Two-Port Circuits

(2) Parallel connection



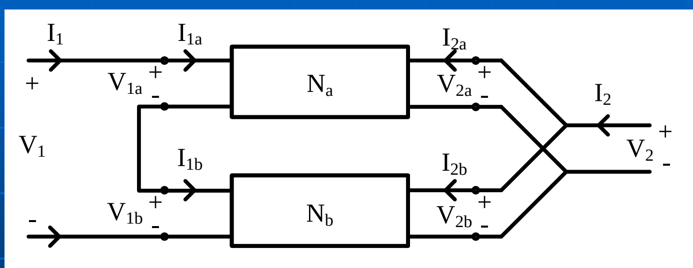
$I_1 = I_{1a} + I_{1b}$ $V_1 = V_{1a} = V_{1b}$

$I_2 = I_{2a} + I_{2b}$ $V_2 = V_{2a} = V_{2b}$

$\therefore \ [Y] = [Y_a] + [Y_b]$

5.5 Interconnected Two-Port Circuits

(3) Series-parallel connection



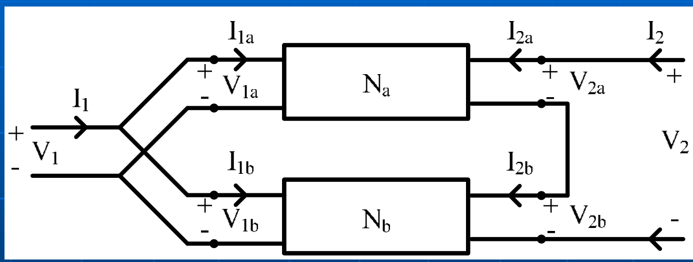
$I_1 = I_{1a} = I_{1b}$ $V_1 = V_{1a} + V_{1b}$

$I_2 = I_{2a} + I_{2b}$ $V_2 = V_{2a} = V_{2b}$

$\therefore \ [h] = [h_a] + [h_b]$

5.5 Interconnected Two-Port Circuits

(4) Parallel-series connection



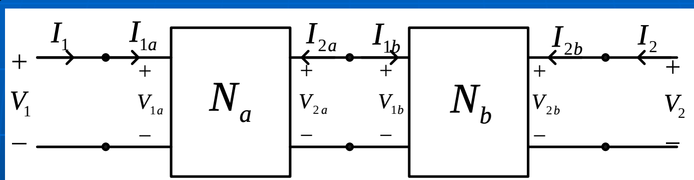
$I_1 = I_{1a} + I_{1b}$ $\quad$ $V_1 = V_{1a} = V_{1b}$

$I_2 = I_{2a} = I_{2b}$ $\quad$ $V_2 = V_{2a} + V_{2b}$

$$\therefore \quad [g] = [g_a] + [g_b]$$

5.5 Interconnected Two-Port Circuits

(5) Cascade connection



$V_1 = V_{1a}$ $\quad$ $I_1 = I_{1a}$

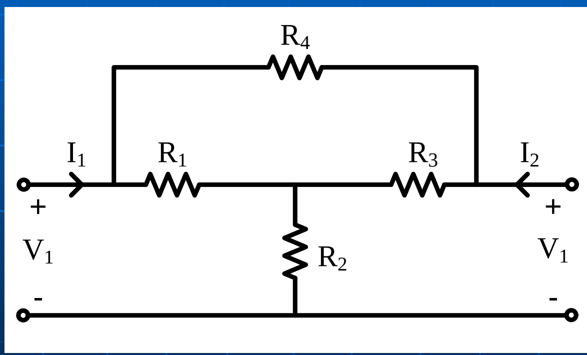
$V_{2a} = V_{1b}$ $\quad$ $I_{1b} = -I_{2a}$

$V_2 = V_{2b}$ $\quad$ $I_2 = I_{2b}$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} a_{11}' & a_{12}' \\ a_{21}' & a_{22}' \end{bmatrix} + \begin{bmatrix} a_{11}'' & a_{12}'' \\ a_{21}'' & a_{22}'' \end{bmatrix}$$

5.5 Interconnected Two-Port Circuits

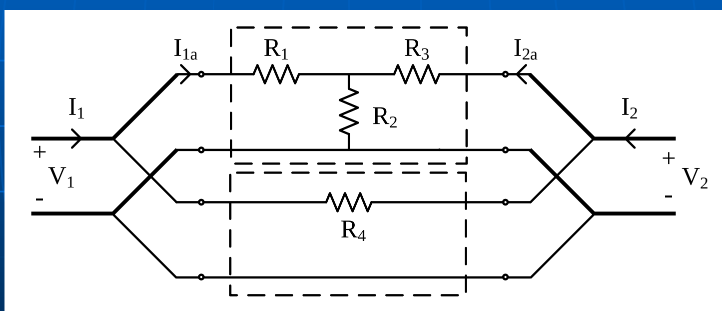
Example 11 : Find the [Z] and [Y] parameters of the following two port network.



5.5 Interconnected Two-Port Circuits

Example 11 : (cont.)

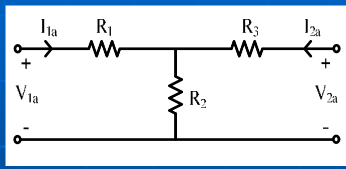
For [y] parameters :



5.5 Interconnected Two-Port Circuits

Example 11 : (cont.)

$$\begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix}$$

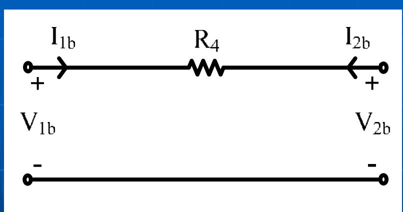


$$\begin{cases} \dfrac{1}{y_{11}} = R_1 + (R_2 \| R_3) = R_1 + \dfrac{R_2 R_3}{R_2 + R_3} \\ \dfrac{1}{y_{22}} = R_3 + (R_1 \| R_2) = R_3 + \dfrac{R_1 R_2}{R_1 + R_2} \end{cases}$$

$$y_{12} = y_{21} = \frac{I_{1a}}{V_2} = -V_2 y_{22} \times \frac{R_2}{R_1+R_2} \frac{1}{V_2} = -y_{22} \times \frac{R_2}{R_1+R_2}$$

5.5 Interconnected Two-Port Circuits

Example 11 : (cont.)



$$\begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix}$$

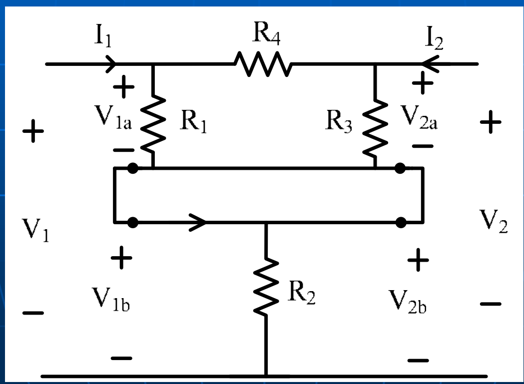
$$\begin{cases} y_{11} = \dfrac{1}{R_4} = y_{22} \\ y_{12} = y_{21} = \dfrac{I_1}{V_2} = -\dfrac{1}{R_4} \end{cases}$$

$$\therefore \; [y] = [y_a] + [y_b]$$

5.5 Interconnected Two-Port Circuits

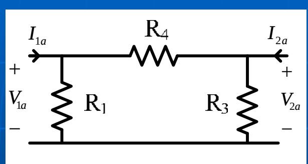
Example 11 : (cont.)

For [z] parameters :

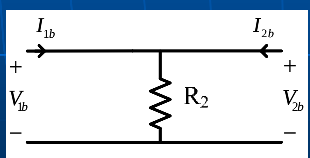


5.5 Interconnected Two-Port Circuits

Example 11 : (cont.)



(a)



(b)

$$Z_{11} = R_1 \parallel (R_3 + R_4)$$

$$Z_{22} = R_3 \parallel (R_1 + R_4)$$

$$Z_{12} = Z_{21} = Z_{22} \times \frac{R_1}{R_1 + R_4}$$

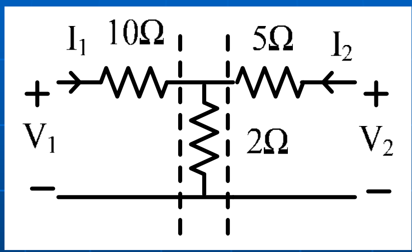
$$Z_{11} = Z_{22} = R_2$$

$$Z_{12} = Z_{21} = R_2$$

$$\therefore \; [Z] = [Z_a] + [Z_b]$$

5.5 Interconnected Two-Port Circuits

Example 12 : Find the a parameters.

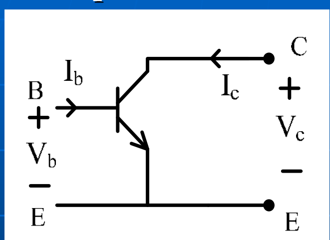


$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & 10 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

5.5 Interconnected Two-Port Circuits

Example 13 : A common emitter circuit



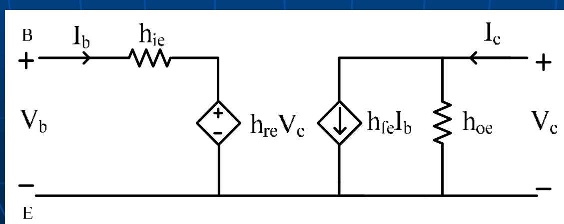
$$\begin{bmatrix} V_b \\ I_c \end{bmatrix} = \begin{bmatrix} h_{ie} & h_{re} \\ h_{fe} & h_{oe} \end{bmatrix} \begin{bmatrix} I_b \\ V_c \end{bmatrix}$$

h_{ie}:base input impedence

h_{re}:reverse voltage gain

h_{fe}:forward current gain

h_{oe}:output admittance



Summary

- Objective 1 : Understand the definition of 6 sets of two port parameters.
- Objective 2 : Be able to find any set of two-port parameters.
- Objective 3 : Be able to analyze a terminated two-port circuit.

Summary

- Objective 4 : Understand the reciprocal theorem for two-port circuits.
- Objective 5 : Know how to analyze an interconnected two-port circuits.

Summary

- Problem : 18.4
  18.8
  18.11
  18.18
  18.37
  18.38
- Due within one week.