CHAPTER 4B

CIRCUIT THEOREMS

CONTENTS

4.6 Superposition Theorem
4.7 Thevenin’s Theorem
4.8 Norton’s Theorem
4.9 Source Transformation
4.10 Maximum Power Transfer Theorem
The relationship $f(x)$ between cause $x$ and effect $y$ is linear if $f(\cdot)$ is both additive and homogeneous.

**Definition of additive property:**

If $f(x_1)=y_1$, $f(x_2)=y_2$ then $f(x_1+x_2)=y_1+y_2$.

**Definition of homogeneous property:**

If $f(x)=y$ and $\alpha$ is a real number then $f(\alpha x)=\alpha y$.

---

**Example 4.6.1**

Assume $I_0 = 1$ A and use linearity to find the actual value of $I_0$ in the circuit in figure.

---

\[ I_0 = 15 \text{ A} \]

\[ 6 \Omega \]

\[ 2 \text{ V} \]

\[ 2 \Omega \]

\[ 1 \text{ V} \]

\[ 3 \Omega \]

\[ 7 \Omega \]

\[ 4 \Omega \]

\[ 5 \Omega \]
4.6 Superposition Theorem

For a linear circuit $N$ consisting of $n$ inputs, namely $u_1, u_2, \ldots, u_n$, then the output $y$ can be calculated as the sum of its components:

$$y = y_1 + y_2 + \ldots + y_n$$

where

$$y_i = f(u_i), \quad i = 1, 2, \ldots, n$$

If $I_o = 1A$, then $V_i = (3+5)I_o = 8V$

$$I_1 = \frac{V_i}{4} = 2A, \quad I_2 = I_1 + I_o = 3A$$

$$V_3 = V_i + 2I_2 = 8 + 6 = 14V, \quad I_3 = \frac{V_3}{7} = 2A$$

$$I_4 = I_3 + I_2 = 5A \Rightarrow I_S = 5A$$

$$I_o = 1A \rightarrow I_s = 5A, \quad I_o = 3A \rightarrow I_s = 15A$$
4.6 Superposition Theorem

Proof: Consider the nodal equation of the corresponding circuit for the basic case as an example:

\[
\begin{bmatrix}
G_{11} & G_{12} & L & G_{1n} \\
G_{21} & G_{22} & L & G_{2n} \\
M & O & M & M \\
G_{n1} & G_{n2} & L & G_{nn}
\end{bmatrix}
\begin{bmatrix}
e_1 \\
e_2 \\
e_n
\end{bmatrix}
=
\begin{bmatrix}
I_{1s} \\
I_{2s} \\
M
\end{bmatrix}
= 
\begin{bmatrix}
I_{ns}
\end{bmatrix}
\]  

Let 
\[G_k = [G_{k1} \ G_{k2} \cdots G_{kn}]^T\]

Then 
\[[G] = [G_1 \ G_2 \cdots G_n] \]

C.T. Pan

4.6 Superposition Theorem

Cramer’s Rule for solving \(Ax = b\)

Take \(n=3\) as an example.

\[
\begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}
=
\begin{bmatrix}
b_1 \\
b_2 \\
b_3
\end{bmatrix}
\]

Let 
\[\det A = \triangle \neq 0\]
4.6 Superposition Theorem

Then

\[
x_1 = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{\Delta}
\]

\[
x_2 = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{\Delta}
\]

\[
x_3 = \frac{\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{\Delta}
\]

4.6 Superposition Theorem

Suppose that the kth nodal voltage \( e_k \) is to be found.

Then from Cramer’s rule one has

\[
e_k = \frac{\det [G, G_I, I_f, L, G_n]}{\det [G]}
\]

\[
e_k = \sum_{j=1}^{n} \frac{\Delta_{jk}}{\Delta} I_{js}
\]

where \( \Delta \) \( \odot \) \( \det [G] \)

\[
\therefore e_k = e_{k1} + e_{k2} + L \ L \ L + e_{kn}
\]
4.6 Superposition Theorem

where

\[ e_{kl} = \frac{\Delta_{lk}}{\Delta} I_{1s}, \text{ due to } I_{1s} \text{ only} \]

\[ \vdots \]

\[ e_{kn} = \frac{\Delta_{nk}}{\Delta} I_{ns}, \text{ due to } I_{ns} \text{ only} \]

---

Example 4.6.2

Find \( e_2 = ? \)

Nodal Equation

\[
\begin{pmatrix}
G_1+G_4+G_6 & -G_4 & -G_6 \\
-G_4 & G_2+G_3+G_5 & -G_5 \\
-G_6 & -G_5 & G_3+G_5+G_6
\end{pmatrix}
\begin{pmatrix}
e_1 \\
e_2 \\
e_3
\end{pmatrix}
= 
\begin{pmatrix}
I_{1s} \\
I_{2s} \\
I_{3s}
\end{pmatrix}
\]
4.6 Superposition Theorem

By using Cramer’s rule

\[
\begin{align*}
\det \begin{pmatrix}
G_1 + G_4 + G_6 & I_{1S} & -G_6 \\
-G_4 & I_{2S} & -G_5 \\
-G_6 & I_{3S} & G_3 + G_5 + G_6
\end{pmatrix}
\end{align*}
\]

\[
e_2 = \frac{\Delta_{12}}{\Delta} I_{1S} + \frac{\Delta_{22}}{\Delta} I_{2S} + \frac{\Delta_{32}}{\Delta} I_{3S}
\]

\[
e_2 = e_{21} + e_{22} + e_{23}
\]

4.6 Superposition Theorem

Where \(e_{21}\) is due to \(I_{1S}\) only, \(I_{2S} = I_{3S} = 0\)
4.6 Superposition Theorem

\[
\begin{array}{ccc|c}
G_1+G_4+G_6 & -G_4 & -G_6 & e_{11} \\
-G_4 & G_5+G_4+G_5 & -G_5 & e_{21} = I_{1S} \\
-G_6 & -G_5 & G_3+G_5+G_6 & e_{31} = 0
\end{array}
\]

\[
det \begin{pmatrix}
G_1 + G_4 + G_6 & I_{1S} & -G_6 \\
-G_4 & 0 & -G_5 \\
-G_6 & 0 & G_3 + G_5 + G_6
\end{pmatrix} = \frac{\Delta_{12}}{\Delta} I_{1S}, \text{ due to } I_{1S} \text{ only}
\]

4.6 Superposition Theorem

Similarly

**Duo to** \( I_{2S} \) **only**

\[
I_{1S} = I_{3S} = 0
\]

**Duo to** \( I_{3S} \) **only**

\[
I_{1S} = I_{2S} = 0
\]
4.7 Thevenin’s Theorem

In high school, one finds the equivalent resistance of a two terminal resistive circuit without sources.

Now, we will find the equivalent circuit for two terminal resistive circuit with sources.

Thevenin’s theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source $V_{TH}$ in series with a resistor $R_{TH}$ where $V_{TH}$ is the open circuit voltage at the terminals and $R_{TH}$ is the input or equivalent resistance at the terminals when the independent sources are turned off.
4.7 Thevenin’s Theorem

Equivalent circuit: same voltage-current relation at the terminals.

\[ V_{TH} = V_{OC} \]: Open circuit voltage at a-b

\[ V_{TH} = V_{OC} \]
4.7 Thevenin’s Theorem

\[ R_{TH} = R_{IN} \]: input resistance of the dead circuit

Turn off all independent sources

CASE 1

If the network has no dependent sources:

- Turn off all independent sources.
- \( R_{TH} \): input resistance of the network looking into a-b terminals
4.7 Thevenin’s Theorem

**CASE 2**

If the network has dependent sources
- Turn off all independent sources.
- Apply a voltage source \( V_0 \) at a-b

\[
R_{TH} = \frac{V_0}{I_0}
\]

Circuit with all independent sources set equal to zero

If \( R_{TH} < 0 \), the circuit is supplying power.

- Alternatively, apply a current source \( I_0 \) at a-b
4.7 Thevenin’s Theorem

Simplified circuit

\[ I_L = \frac{V_{TH}}{R_{TH} + R_L} \]
\[ V_L = R_L I_L = \frac{R_L}{R_{TH} + R_L} V_{TH} \]

Voltage divider

Proof: Consider the following linear two terminal circuit consisting of \( n+1 \) nodes and choose terminal \( b \) as datum node and terminal \( a \) as node \( n \).

\[
\begin{bmatrix}
G_{11} & K & G_{1n} \\
M & O & M \\
G_{n1} & L & G_{nn}
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
V_n
\end{bmatrix}
=
\begin{bmatrix}
I_{1s} \\
I_{2s} \\
I_{ns}
\end{bmatrix}
\]
4.7 Thevenin’s Theorem

Then nodal voltage $V_n$ when a-b terminals are open can be found by using Cramer’s rule.

$$V_n = \frac{1}{\Delta} \sum_{k=1}^{n} \Delta_{kn} I_{ks} \quad L \quad L \quad L \quad (A)$$

$\Delta$ is the determinant of $[G]$ matrix

$\Delta_{kn}$ is the corresponding cofactor of $G_{kn}$

Now connect an external resistance $R_o$ to a-b terminals.

The new nodal voltages will be changed to $e_1, e_2, \ldots, e_n$ respectively.

C.T. Pan 27

4.7 Thevenin’s Theorem

Nodal equation

$$\begin{pmatrix}
G_{11} & K & G_{1n} + 0 \\
M & G_{2n} + 0 \\
M & M \\
G_{nl} & L & G_{m} + \frac{1}{R_n}
\end{pmatrix}
\begin{pmatrix}
e_1 \\
e_2 \\
e_n
\end{pmatrix} =
\begin{pmatrix}
I_{1s} \\
I_{2s} \\
I_{ns}
\end{pmatrix} \quad \ldots \ldots \quad (B)$$

C.T. Pan 28
4.7 Thevenin’s Theorem

Note that

\[
\begin{vmatrix}
G_{11} & K & G_{1n} + 0 \\
M & G_{2n} + 0 & \\
G_{n1} & L & G_{n} + \frac{1}{R_o}
\end{vmatrix} = \det(G) + \det\left(\begin{array}{ccc}
G_{11} & K & 0 \\
G_{21} & 0 & \\
G_{n1} & L & \frac{1}{R_o}
\end{array}\right)
\]

\[
= \Delta + \frac{1}{R_o} \Delta_{mn}
\]

4.7 Thevenin’s Theorem

Hence, \(e_n\) can be obtained as follows.

\[
e_n = \frac{\det\left(\begin{array}{ccc}
G_{11} & K & I_{1s} \\
M & O & M \\
G_{n1} & L & I_{ns}
\end{array}\right)}{\Delta + \frac{1}{R_o} \Delta_{ss}} = \frac{\sum_{s=1}^{n} \Delta_{m} I_{ks}}{\Delta + \frac{1}{R_o} \Delta_{ss}} = \frac{1}{\Delta} \sum_{s=1}^{n} \Delta_{m} I_{ks} = \frac{1}{\Delta + \frac{1}{R_o} \Delta_{ss}} = \frac{R_o}{R_o + R_{TH}} V_n
\]

where \(R_{TH} \bigotimes \frac{\Delta_{nn}}{\Delta}\)
In other words, the linear circuit looking into terminals a-b can be replaced by an equivalent circuit consisting of a voltage source $V_{TH}$ in series with an equivalent resistance $R_{TH}$, where $V_{TH}$ is the open circuit voltage $V_n$ and $R_{TH} = \frac{\Delta_{m}}{\Delta}$.

### Example 4.7.1

Consider the circuit shown in the image. The voltage source $V_x$ is known, and the current source $I_a$ is applied. The goal is to find the equivalent circuit that represents the original circuit when looking into terminals a-b.

1. Identify the circuit elements connected between a and b.
2. Calculate the equivalent resistance $R_{TH}$ using $R_{TH} = \frac{\Delta_{m}}{\Delta}$.
3. Use the Thevenin's theorem to find the voltage $V_{TH}$ across the terminals a-b.

By following these steps, the equivalent circuit can be determined, and the analysis can proceed accordingly.
Example 4.7.1 (cont.)

Find open circuit voltage $V_2$

\[
\begin{pmatrix}
2 + 4 & -2 \\
-2 & 2 + 6
\end{pmatrix}
\begin{pmatrix}
V_1 \\
V_2
\end{pmatrix} = 
\begin{pmatrix}
5 - 2V_x \\
2V_x
\end{pmatrix}
\]

$2V_x = 2V_1$

\[
\begin{pmatrix}
2 + 4 + 2 & -2 \\
-2 - 2 & 2 + 6
\end{pmatrix}
\begin{pmatrix}
V_1 \\
V_2
\end{pmatrix} = 
\begin{pmatrix}
5 \\
0
\end{pmatrix}
\]

$\Delta = \det\begin{pmatrix}
8 & -2 \\
-4 & 8
\end{pmatrix} = 64 - 8 = 56$

\[
\det\begin{pmatrix}
8 & 5 \\
-4 & 0
\end{pmatrix} = \frac{20}{56} = \frac{5}{14} V = V_{TH}
\]

\[
R_{TH} = \frac{\Delta_{12}}{\Delta} = \frac{8}{56} = \frac{1}{7} \Omega
\]

\[\therefore\text{Ans.}\]

\[
\frac{1}{7} \Omega
\]

\[
\frac{5}{14} V
\]
4.7 Thevenin’s Theorem

Example 4.7.2

By voltage divider principle:
open circuit voltage $V_{TH} = 10V$
Let independent source be zero

Example 4.7.3

Find the Thevenin’s equivalent circuit of the circuit shown below, to the left of the terminals a-b. Then find the current through $R_L = 6, 16, \text{ and } 36 \Omega$. 
4.7 Thevenin’s Theorem

Example 4.7.3 (cont.)

\[ R_{TH} : \text{32V voltage source} \rightarrow \text{short} \]
\[ R_{TH} : \text{2A current source} \rightarrow \text{open} \]
\[ R_{TH} = 4 \times 12 + 1 = \frac{4 \times 12}{16} + 1 = 4 \Omega \]

\[ V_{TH} : \]
Mesh analysis
\[-32 + 4i_1 + 12(i_1 - i_2) = 0, i_2 = -2A \]
\[ \therefore i_1 = 0.5A \]
\[ V_{TH} = 12(i_1 - i_2) = 12(0.5 + 2.0) = 30V \]
4.7 Thevenin’s Theorem

Example 4.7.3 (cont.)

To get $i_L$:

$$i_L = \frac{V_{TH}}{R_{TH} + R_L} = \frac{30}{4 + R_L}$$

- $R_L = 6 \rightarrow I_L = 30/10 = 3\text{A}$
- $R_L = 16 \rightarrow I_L = 30/20 = 1.5\text{A}$
- $R_L = 36 \rightarrow I_L = 30/40 = 0.75\text{A}$

4.7 Thevenin’s Theorem

Example 4.7.4

Find the Thevenin’s equivalent of the following circuit with terminals a-b.

[Diagram of a circuit with a 5 A current source, a voltage source $V_x$, and resistors $2\Omega$, $4\Omega$, $2\Omega$, and $6\Omega$.]
Example 4.7.4 (cont.)

(independent + dependent source case)

To find $R_{TH}$ from Fig.(a)

- independent source $\rightarrow 0$
- dependent source $\rightarrow$ unchanged

Apply

\[
\begin{align*}
    v_o &= 1V, \\
    R_{TH} &= \frac{v_o}{i_o} = \frac{1}{i_o}
\end{align*}
\]

4.7 Thevenin’s Theorem

Example 4.7.4 (cont.)

For loop 1,

\[
-2v_x + 2(i_1 - i_2) = 0 \quad \text{or} \quad v_x = i_1 - i_2
\]

But

\[
-4i_2 = v_x = i_1 - i_2
\]

\[
\therefore i_1 = -3i_2
\]
4.7 Thevenin’s Theorem

Example 4.7.4 (cont.)

Loop 2 and 3:

\[ 4i_2 + 2(i_2 - i_1) + 6(i_2 - i_3) = 0 \]
\[ 6(i_3 - i_2) + 2i_3 + 1 = 0 \]

Solving these equations gives

\[ i_3 = -\frac{1}{6} \]
\[ i_i = -i_3 = \frac{1}{6} \]
\[ R_{TH} = \frac{1V}{i_i} = 6\Omega \]

To find \( V_{TH} \) from Fig. (b)

Mesh analysis

\[ i_4 = 5 \]
\[ -2v_x + 2(i_3 - i_2) = 0 \Rightarrow v_x = i_3 - i_2 \]
\[ 4(i_2 - i_i) + 2(i_2 - i_3) + 6i_2 = 0 \Rightarrow 12i_2 - 4i_4 - 2i_3 = 0 \]
**4.7 Thevenin’s Theorem**

Example 4.7.4 (cont.)

But \(4(i_1 - i_2) = V_1\)

\[\therefore i_2 = \frac{10}{3} \text{.}\]

\[V_{TH} = v_{oc} = 6i_2 = 20 \text{V}\]

Determine the Thevenin’s equivalent circuit:

Solution:

(dependent source only)

\[V_{TH} = 0, \quad R_{TH} = \frac{V_o}{i_o}\]

Nodal analysis

\[i_o + i_x = 2i_x + \frac{v_o}{4}\]
4.7 Thevenin’s Theorem

Example 4.7.5 (cont.)

But

\[ i_x = \frac{0 - v_o}{2} = -\frac{v_o}{2} \]
\[ i_o = i_x + \frac{v_o}{4} = -\frac{v_o}{2} + \frac{v_o}{4} = -\frac{v_o}{4} \]

or \[ v_o = -4i_o \]

Thus

\[ R_{TH} = \frac{v_o}{i_o} = -4\Omega \] : Supplying Power!

4.8 Norton’s Theorem

Norton’s theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source \( I_N \) in parallel with a resistor \( R_N \) where \( I_N \) is the short-circuit current through the terminals and \( R_N \) is the input or equivalent resistance at the terminals when the independent sources are turned off.
4.8 Norton’s Theorem

Proof:
By using Mesh Analysis as an example, assume the linear two-terminal circuit is a planar circuit and there are \( n \) meshes when \( a \) and \( b \) terminals are short circuited.
4.8 Norton’s Theorem

Mesh equation for case 1 as an example

\[
\begin{pmatrix}
R_{11} & \cdots & R_{1n} \\
M & \cdots & M \\
M & \cdots & M \\
R_{n1} & \cdots & R_{nn}
\end{pmatrix}
\begin{pmatrix}
I_1 \\
I_2 \\
I_n
\end{pmatrix} =
\begin{pmatrix}
V_{15} \\
V_{25} \\
M
\end{pmatrix}
\]

Hence the short circuit current

\[I_n = \frac{1}{\Delta} \sum_{k=1}^{n} \Delta_{kn} V_{ks}\]

where \(\Delta = \det[R_{ik}]\)

\(\Delta_{kn}\) is the cofactor of \(R_{kn}\)

4.8 Norton’s Theorem

Now connect an external resistance \(R_o\) to \(a, b\) terminals, then all the mesh currents will be changed to \(J_1, J_2, \cdots J_n\) respectively.

\[
\begin{pmatrix}
R_{11} & \cdots & R_{1n} + 0 \\
M & \cdots & M \\
M & \cdots & M \\
R_{n1} & \cdots & R_{nn} + R_o
\end{pmatrix}
\begin{pmatrix}
J_1 \\
J_2 \\
J_n
\end{pmatrix} =
\begin{pmatrix}
V_{15} \\
V_{25} \\
M
\end{pmatrix}
\]

Note that

\[
\det\begin{pmatrix}
R_{11} & \cdots & R_{1n} + 0 \\
M & \cdots & M \\
M & \cdots & M \\
R_{n1} & \cdots & R_{nn} + R_o
\end{pmatrix} = \Delta + \det\begin{pmatrix}
R_{11} & K & 0 \\
M & \cdots & M \\
M & \cdots & M \\
R_{n1} & \cdots & R_o
\end{pmatrix}
\]

\(= \Delta + R_o \Delta_{nn}\)
4.8 Norton’s Theorem

Hence, one has

\[
J_n = \frac{\det \begin{pmatrix} R_{11} & \cdots & V_{1r} \\ M & O & M \\ R_{n1} & L & V_{ns} \end{pmatrix}}{\Delta + R_o \Delta_{mn}} = \frac{\sum_{k=1}^{n} \Delta_{kn} V_{ks}}{\Delta + R_o \Delta_{mn}} = \frac{1}{\Delta} \sum_{k=1}^{n} \Delta_{kn} V_{ks} = \frac{1}{1 + R_o \Delta_{mn}} \Delta
\]

\[
\Delta_{nm} \Delta = \Delta
\]

4.8 Norton’s Theorem

\[
I_n = \frac{I_N}{1 + R_o \frac{\Delta_{mn}}{\Delta}} = \frac{R_N}{R_o + R_N} I_n
\]

where \( R_N = \frac{\Delta}{\Delta_{mn}} \), \( I_N = I_n \)
4.8 Norton’s Theorem

Example 4.8.1 (cont.)

By using the above formula

\[ \begin{vmatrix} 3+3 & -3 & -3 \\ -3 & 3+3+4 & -3 \\ -3 & -3 & 3+3 \end{vmatrix} \]

\[ I_1 = \frac{1}{108} \begin{vmatrix} 6 & -3 & 10 \\ -3 & 10 & 0 \\ -3 & -3 & 0 \end{vmatrix} = \frac{10}{108} \begin{vmatrix} 39 \end{vmatrix} = \frac{390}{108} = \frac{65}{18} A = I_N \]

\[ R_N = \frac{\Delta}{\Delta_{33}} = \frac{108}{60-9} = \frac{36}{17} \Omega \]
4.8 Norton’s Theorem

Example 4.8.2

Find the Norton equivalent circuit of the following circuit:

To find $R_N$ from Fig.(a):

$$R_N = 5 \|(8 + 4 + 8)$$
$$= 5 \| 20 = \frac{20 \times 5}{25} = 4\Omega$$
4.8 Norton’s Theorem

Example 4.8.2 (cont.)

To find $I_N$ from Fig. (b)

short-circuit terminal $a$ and $b$

Mesh Analysis:

$i_1 = 2A$

$20i_2 - 4i_1 - 12 = 0$

$\therefore i_2 = 1A = I_N$

Alternative method for $I_N$: $I_N = \frac{V_{TH}}{R_{TH}}$

$V_{TH}$: open-circuit voltage across terminals $a$ and $b$

Mesh analysis:

$i_3 = 2A$, $25i_4 - 4i_3 - 12 = 0$

$\therefore i_4 = 0.8A$

$\therefore V_{oc} = V_{HH} = 5i_4 = 4V$
4.8 Norton’s Theorem

Example 4.8.2 (cont.)

Hence, \[ I_N = \frac{V_{TH}}{R_{TH}} = \frac{4}{4} = 1 \text{A} \]

\[ \text{a} \quad 1 \text{A} \quad 4 \Omega \quad \text{b} \]

Example 4.8.3

Using Norton’s theorem, find \( R_N \) and \( I_N \) of the following circuit.
4.8 Norton’s Theorem

Example 4.8.3 (cont.)

To find $R_N$ from Fig.(a)

\[ i_o = \frac{v_o}{5} = \frac{1}{5} = 0.2 \text{ A} \]

\[ \therefore R_N = \frac{v_o}{i_o} = \frac{1}{0.2} = 5\Omega \]

To find $I_N$ from Fig.(b)

\[ i_x = \frac{10}{4} = 2.5 \text{ A} \]

\[ I_N = \frac{10\text{V}}{5\Omega} + 2i_x \]

\[ = \frac{10}{5} + 2(2.5) = 7\text{ A} \]

\[ \therefore I_N = 7\text{ A} \]
4.9 Source Transformation

The current through resistor R can be obtained as follows:

\[ i = \frac{V_s - v}{R} = \frac{V_s}{R} - \frac{v}{R} \]

From KCL, one can obtain the following equivalent circuit:

\[ where \quad I_s \@ \frac{V_s}{R} \]
4.9 Source Transformation

The voltage across resistor $R$ can be obtained as follows:

$$v = (I_s - i)R = I_s R - iR \triangleleft V_s - IR$$

From KVL, one can obtain the following equivalent circuit:

$$V_s \triangleleft R \triangleleft I_s$$
4.9 Source Transformation

Example 4.9.1

\[ 5\Omega \]

\[ 20\text{V} \rightarrow 4\text{A} \]

\[ 10\text{A} \]

\[ 3\Omega \]

\[ 30\text{V} \]

Example 4.9.2

Find the Thevenin’s equivalent

\[ 4\Omega \]

\[ 3\Omega \]

\[ 20\text{V} \rightarrow 5\text{A} \]

\[ 5\text{A} \]

\[ 4\Omega \]

\[ 5\text{A} \]
Example 4.9.2 (cont.)

Given a linear resistive circuit shown as above, find the value of \( R_L \) that permits maximum power delivery to \( R_L \).

\[ V_{TH} = 40V \]
\[ R_{TH} = 7\Omega \]

4.10 Maximum Power Transfer Theorem

Problem: Given a linear resistive circuit \( N \) shown as above, find the value of \( R_L \) that permits maximum power delivery to \( R_L \).
4.10 Maximum Power Transfer Theorem

Solution: First, replace N with its Thevenin equivalent circuit.

\[ i = \frac{V_{TH}}{R_{TH} + R_L} \]

\[ P = i^2 R = \left( \frac{V_{TH}}{R_{TH} + R_L} \right)^2 R_L \]

Let \( \frac{dp}{dR_L} = 0 \),

Then \( R_L = R_{TH} \) and \( P_{\text{max}} = \left( \frac{V_{TH}}{2R_L} \right)^2 R_L = \frac{V_{TH}^2}{4R_L} \)
4.10 Maximum Power Transfer Theorem

Example 4.10.1

(a) Find $R_L$ that results in maximum power transferred to $R_L$.
(b) Find the corresponding maximum power delivered to $R_L$, namely $P_{\text{max}}$.
(c) Find the corresponding power delivered by the 360V source, namely $P_s$ and $P_{\text{max}}/P_s$ in percentage.

\[ S \text{olution: (a) } V_{\text{TH}} = \frac{150}{180} (360) = 300V \]
\[ R_{\text{TH}} = \frac{150 \times 30}{180} = 25 \Omega \]
\[ (b) \ P_{\text{max}} = \left( \frac{300}{50} \right)^2 25 = 900W \]
4.10 Maximum Power Transfer Theorem

Solution: (c) \( V_{ab} = \frac{300}{50} \times 25 = 150 \text{V} \)

\( i_s = \frac{(360 - 150)}{30} = -7 \text{A} \)

\( P_s = i_s \times 360 = -2520 \text{W (dissipated)} \)

\( \frac{P_{\text{max}}}{|P_s|} = \frac{900}{2520} = 35.71\% \)

Summary

<table>
<thead>
<tr>
<th>Objective 7</th>
<th>Understand and be able to use superposition theorem.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objective 8</td>
<td>Understand and be able to use Thevenin’s theorem.</td>
</tr>
<tr>
<td>Objective 9</td>
<td>Understand and be able to use Norton’s theorem.</td>
</tr>
</tbody>
</table>
Summary

Objective 10: Understand and be able to use source transform technique.

Objective 11: Know the condition for and be able to find the maximum power transfer.

Problem: 4.60
4.64
4.68
4.77
4.86
4.91

Due within one week.