CHAPTER 4A

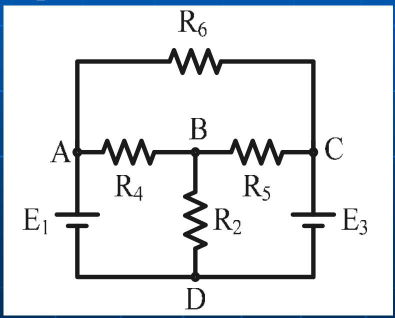
TECHNIQUES OF CIRCUIT ANALYSIS

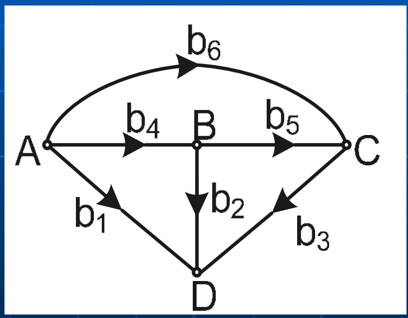
CONTENTS

4.1 Introduction
4.2 The Node-Voltage Method (Nodal Analysis)
4.3 The Mesh-Current Method (Mesh Analysis)
4.4 Fundamental Loop Analysis
4.5 Fundamental Cutset Analysis

4.1 Introduction

A circuit consists of b branches and n nodes.
A direct algebraic approach：2b method
Example.





4.1 Introduction

b=6，n=4　　3 independent equations only (n-1)

KCL：

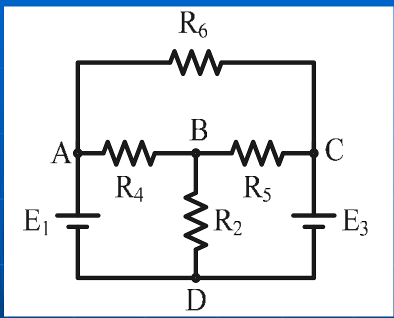
node A：　$I_1+I_4+I_6=0$

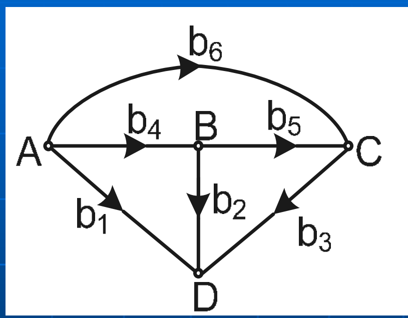
node B：　$-I_4+I_2+I_5=0$

node C：　$I_3-I_5-I_6=0$

node D：　$I_1+I_2+I_3=0$

4.1 Introduction





KVL：

loop ABDA：$V_4+V_2-V_1=0$

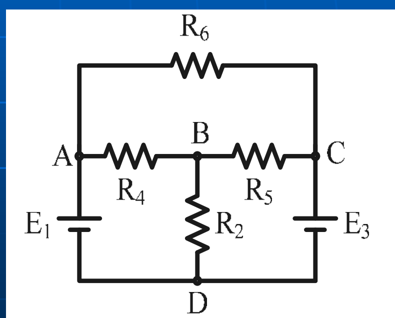
loop BCDB：$V_5+V_3-V_2=0$

loop ACBA：$V_6-V_5-V_4=0$

4.1 Introduction

Component model (b branches)

$V_1 = E_1$, $V_4 = R_4 I_4$,
$V_2 = R_2 I_2$, $V_5 = R_5 I_5$,
$V_3 = E_3$, $V_6 = R_6 I_6$,



4.1 Introduction

Want to find V_k , I_k , K=1,2,3, … ,6

There are 2b unknowns.

number of KCL eqs ： n-1

number of KVL eqs ： b-(n-1)

number of component model ： b

The total number of equations.

$$n-1+[b-(n-1)]+b=2b$$

4.1 Introduction

Just enough information to find the solution.

Problem ： Too many variables !!

It is not efficient !

Question ： Is it possible to find an optimal method with minimum unknowns ?

4.1 Introduction

- Nodal analysis is based on a systematic application of KCL and is a general method.
- Mesh Analysis is based on a systematic application of KVL and can be used for planar circuits only.

4.1 Introduction

- Fundamental loop analysis is based on a systematic application of KVL to the fundamental loops. It requires the definition of tree.
- Fundamental cutset analysis is based on a systematic application of KCL to the fundamental cutsets. It also requires the definition of tree.

4.1 Introduction

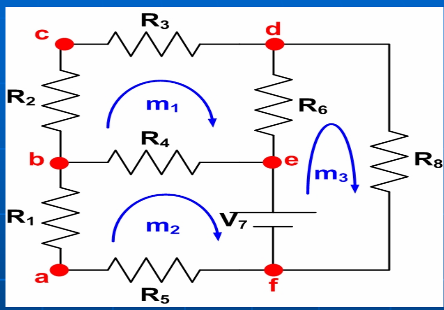
- Node: A point where two or more elements join
- Path: A trace of adjoining basic elements with no elements included more than once
- Branch: A path that connects two nodes
- Essential node: A node where three or more elements join
- Essential branch: A path that connects two essential nodes without passing through an essential node

4.1 Introduction

- Loop: A path whose last node is the same as the starting node
- Mesh: A loop that does not enclose any other loops
- Planar circuit: A circuit that can be drawn on a plane with no crossing branches

4.1 Introduction

- Example 4.1.1



The nodes are { a , b , c , d , e , f }

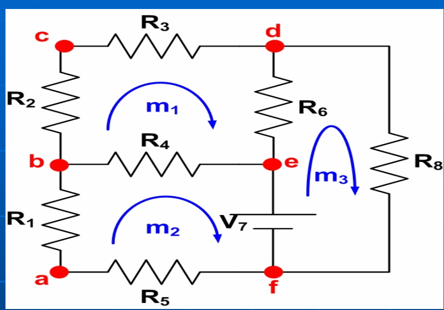
The essential nodes are { b , d , e , f }

The branches are { R_1 , R_2 , R_3 , R_4 , R_5 , R_6 , V_7 , R_8 }

The essential branches are { R_4 , R_6 , V_7 , R_8 , $R_2 - R_3$, $R_1 - R_5$ }

4.1 Introduction

- Example 4.1.1



The meshes are

m_1 : b – c – d – e – b

m_2 : a – b – e – f – a

m_3 : f – e – d – f

4.1 Introduction

A *graph G* is defined as a set: G(B , N , F)

B: set of branches

N: set of nodes

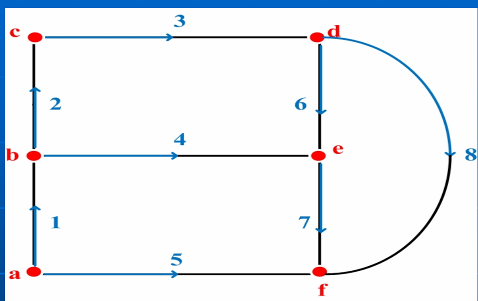
F: a function specifying how a branch is connected.

Oriented (directed) graph $\hat{G}$,
if each branch of G is assigned a direction.

4.1 Introduction

- Example 4.1.2

$\hat{G}(B, N, F)$



$B = \{ b_1 , b_2 , b_3 , b_4 , b_5 , b_6 , b_7 , b_8 \}$
$N = \{ a , b , c , d , e , f \}$
$F = \{ f_1 , f_2 , f_3 , f_4 , f_5 , f_6 , f_7 , f_8 \}$
$f_1 : a \rightarrow b$
$f_2 : b \rightarrow c$

4.1 Introduction

For each oriented graph $\hat{G}$ one can construct a *tree T*.

A tree T of $\hat{G}$ is a connected graph which
(a) contains all the nodes.
(b) without forming any loop.

The branches which do not belong to tree T are called *links.*

Each tree branch defines a unique cutset (or super node) , called fundamental cutset.

4.1 Introduction

Each link together with its unique path connecting the two nodes of the link defines a unique loop , called fundamental loop.

A network with b branches , *n* nodes , and *l* links will satisfy the fundamental theorem of network topology :

$$\underline{b = l + n - 1}$$

4.1 Introduction

Theorem

Given a connected graph G of n nodes and b branches, and a tree T of G

- There is unique path along the tree between any pair of nodes.
- There are n-1 tree branches and b-(n-1) links.

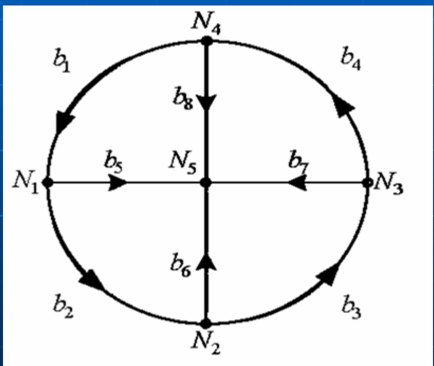
4.1 Introduction

- Every link of T and the unique tree path between its nodes constitute a unique loop, called the fundamental loop associated with the link.
- Every tree branch of T together with some links defines a unique cut set of G, called the fundamental cut set associated with the tree branch.

4.1 Introduction

Example 4.1.3

$\hat{G}(B, N, F)$, b=8 , n=5



$B=\{b_1\ b_2\ b_3\ b_4\ b_5\ b_6\ b_7\ b_8\}$

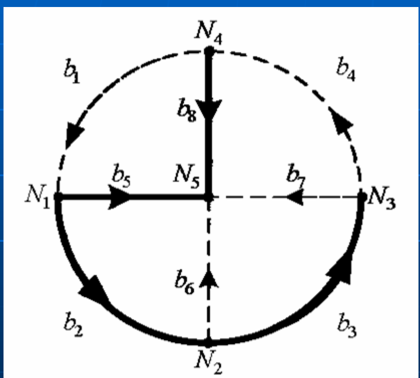
$N=\{N_1\ N_2\ N_3\ N_4\ N_5\}$

$F=[f_1\ f_2\ f_3\ f_4\ f_5\ f_6\ f_7\ f_8]^T$

$f_1: N_4 \rightarrow N_1$
$f_2: N_1 \rightarrow N_2$
$f_3: N_2 \rightarrow N_3$

4.1 Introduction

Example 4.1.4



Fundamental loops :

$T=\{b_2\ b_3\ b_5\ b_8\}$

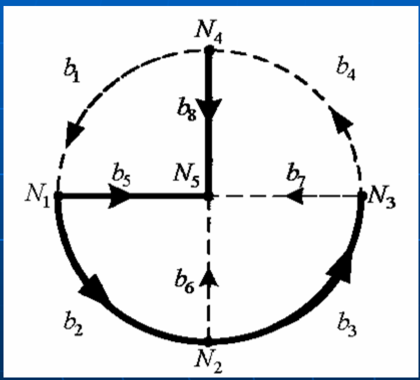
tree branches b_2 , b_3 , b_5 , b_8
no. of tree branches = n-1=4

$L=\{b_1 , b_4 , b_6 , b_7\}$

links b_1 , b_4 , b_6 , b_7 no. of links = b-(n-1)
= 8-(5-1)
= 4

4.1 Introduction

Example 4.1.4



Fundamental loops :

For l_1: $N_1-N_5-N_4-N_1$

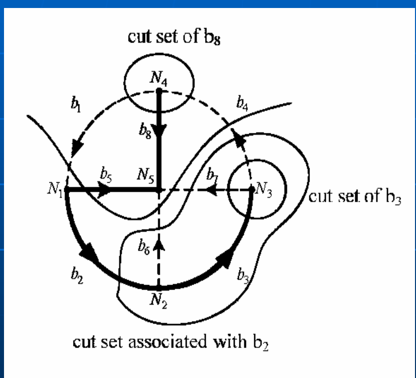
For l_4: $N_4-N_5-N_1-N_2-N_3-N_4$

For l_6: $N_5-N_1-N_2-N_5$

For l_7: $N_5-N_1-N_2-N_3-N_5$

4.1 Introduction

Example 4.1.5



Fundamental cutsets :

Cut set of b_8 = {b_8 , b_1 , b_4}

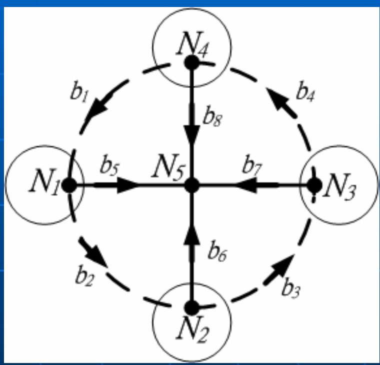
Cut set of b_8 = {b_3 , b_4 , b_7}

Cut set of b_8 = {b_2 , b_6 , b_7 , b_4}

Cut set of b_8 = {b_5 , b_1 , b_6 , b_7 , b_4}

4.1 Introduction

Example 4.1.5



n=5 , b=8

$T_2 = \{b_5 , b_6 , b_7 , b_8\}$

Fundamental cut sets :

Cut set of $b_5 = \{b_1 , b_2 , b_5\}$

Cut set of $b_6 = \{b_2 , b_3 , b_6\}$

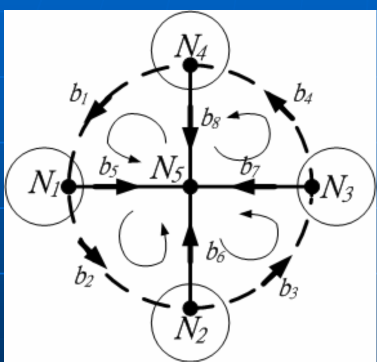
Cut set of $b_7 = \{b_3 , b_4 , b_7\}$

Cut set of $b_8 = \{b_1 , b_4 , b_8\}$

Choose tree branch current direction as reference. Cut only one tree branch.

4.1 Introduction

Example 4.1.5



Set of links = $\{b_1 , b_2 , b_3 , b_4\}$

Fundamental loops :

Loop of $b_1 = N_1 \rightarrow N_5 \rightarrow \underline{N_4 \rightarrow N_1}$

Loop of $b_2 = N_2 \rightarrow N_5 \rightarrow \underline{N_1 \rightarrow N_2}$

Loop of $b_3 = N_3 \rightarrow N_5 \rightarrow \underline{N_2 \rightarrow N_3}$

Loop of $b_4 = N_4 \rightarrow N_5 \rightarrow \underline{N_3 \rightarrow N_4}$

Choose link current as loop current. Contain only one link.

4.2 The Node-Voltage Method

For Linear Resistive Circuits (Example)

- KCL
- KVL
- Component Model (Ohm’s Law)

4.2 The Node-Voltage Method

- Instead of using branch voltages and branch currents as unknowns, one can choose nodal voltages as unknowns. Only n-1 nodal voltages are required.
- Each branch voltage can be obtained from nodal voltages and the corresponding branch currents can be calculated by its component model.
- KVL is automatically satisfied.

4.2 The Node-Voltage Method

- One need only find the independent KCL equations， and the component model is used to express the branch current in terms of nodal voltages.

$$G\underline{e} = \underline{I}_s$$

- Any unknown can be calculated once the nodal voltage vector is solved.

4.2 The Node-Voltage Method

Case 1. Containing current source only

(A) independent current source by inspection

(B) with dependent current source express the controlling parameter in terms of nodal voltages

4.2 The Node-Voltage Method

Case 2. Containing voltage source (VS)

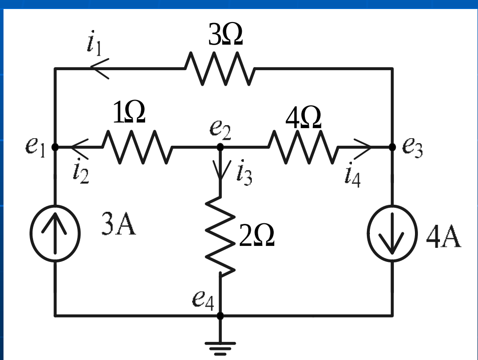
(A) VS contains datum node
trivial case

(B) VS does not contain datum node
by supernode concept

(C) for dependent VS
express the controlling parameter in terms of nodal voltages

4.2 The Node-Voltage Method

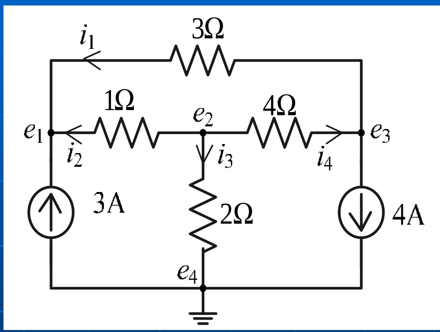
Example 4.2.1. (Case 1A) (Basic Case)



Step 1

Select a node as datum node and choose nodal variables e_1, e_2, e_3 as unknowns

4.2 The Node-Voltage Method



Step 2

Apply KCL to n-1 nodes, use ohm's law to express the branch currents in terms of nodal voltages

$$\sum_{\text{leaving}} i_k = 0$$

$$\text{node 1}: \quad -i_1 - i_2 - 3A = 0$$

$$\text{node 2}: \quad +i_2 + i_3 + i_4 = 0$$

$$\text{node 3}: \quad +i_1 - i_4 + 4A = 0$$

4.2 The Node-Voltage Method

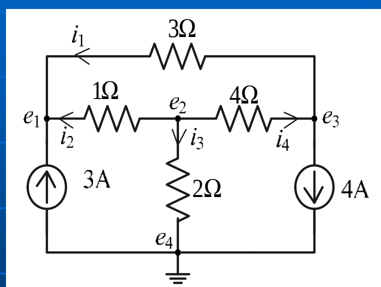
Use ohm's law

$$\text{node 1}: \quad \frac{(e_1 - e_3)}{3} + \frac{(e_1 - e_2)}{1} - 3A = 0$$

$$\text{node 2}: \quad \frac{(e_2 - e_1)}{1} + \frac{(e_2 - 0)}{2} + \frac{(e_2 - e_3)}{4} = 0$$

$$\text{node 3}: \quad +\frac{(e_3 - e_1)}{3} + \frac{(e_3 - e_2)}{4} + 4A = 0$$

$$\begin{pmatrix} 1+\frac{1}{3} & -1 & -\frac{1}{3} \\ -1 & 1+\frac{1}{2}+\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{3} & -\frac{1}{4} & \frac{1}{3}+\frac{1}{4} \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} = \begin{pmatrix} 3A \\ 0 \\ -4A \end{pmatrix}$$



4.2 The Node-Voltage Method

$$\left(G_{jk}\right)\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} = G\,\underline{e} = \begin{pmatrix} I_{s1} \\ I_{s2} \\ I_{s3} \end{pmatrix} = \underline{I_s}$$

G : conduction matrix

$\mathbf{G_{kk}}$: sum of the conductances connected to node k

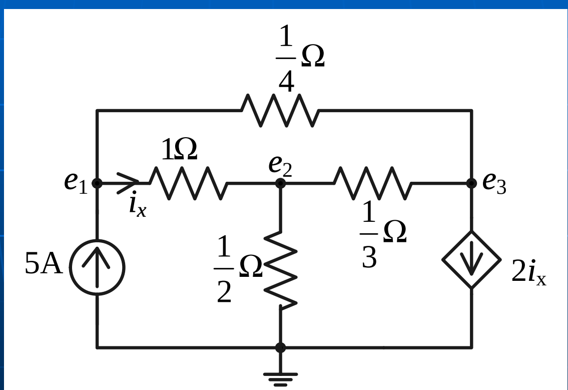
$G_{kj}=G_{jk}$: negative of the sum of conductances directly connecting nodes k and j , $k \neq j$

e_k : nodal voltage of the kth node

I_{sk} : sum of independent current sources entering to node k

4.2 The Node-Voltage Method

Example 4.2.2. (Case 1B)



4.2 The Node-Voltage Method

Step 1 Same as Example 1

$$\begin{pmatrix} 1+4 & -1 & -4 \\ -1 & 1+2+3 & -3 \\ -4 & -3 & 3+4 \end{pmatrix}\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} = \begin{pmatrix} 5A \\ 0 \\ -2i_x \end{pmatrix}$$

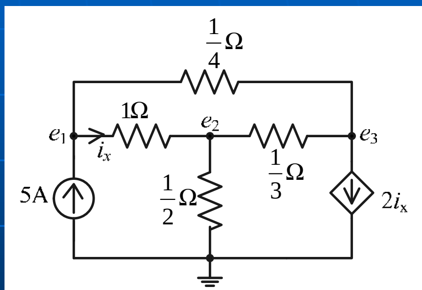
Step 2 express $-2i_x$ in terms of nodal voltages

$$-2i_x = -2\frac{(e_1 - e_2)}{1}$$

4.2 The Node-Voltage Method

Step 3 Move the unknowns to the left hand side of the equation

$$\begin{pmatrix} 5 & -1 & -4 \\ -1 & 6 & -3 \\ -4+2 & -3-2 & 3+4 \end{pmatrix}\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix}$$

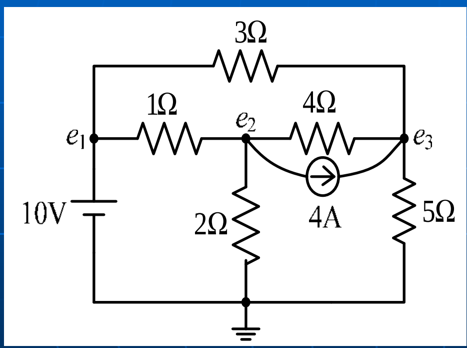


$$G\underline{e} = \underline{I_s}$$

Note : G matrix is no longer symmetric.

4.2 The Node-Voltage Method

Example 4.2.3. (Case 2A) (trivial case)



Step 1

Select datum node .

choose nodal voltages e_1 , e_2 , e_3

Step 2

KCL eqs.

e_1=10V , already known

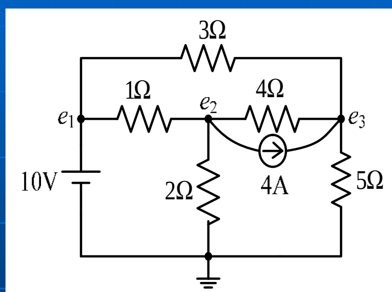
Need 2 KCL eqs. only!

4.2 The Node-Voltage Method

node 1 : $e_1 = 10V$

node 2 : $\dfrac{e_2 - 10}{1} + \dfrac{e_2 - 0}{2} + \dfrac{e_2 - e_3}{4} = -4A$

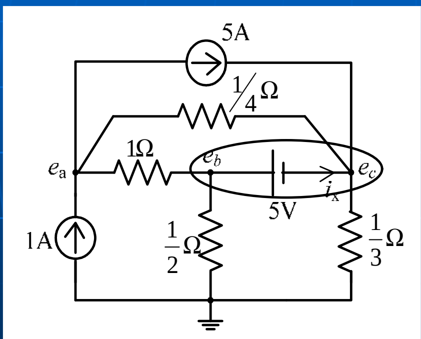
node 3 : $\dfrac{e_3 - 10}{3} + \dfrac{e_3 - e_2}{4} + \dfrac{e_3}{5} = 4A$



$$\begin{pmatrix} 1 + \frac{1}{2} + \frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{3} + \frac{1}{4} + \frac{1}{5} \end{pmatrix} \begin{pmatrix} e_2 \\ e_3 \end{pmatrix} = \begin{pmatrix} 10 - 4 \\ \frac{10}{3} + 4 \end{pmatrix}$$

4.2 The Node-Voltage Method

Example 4.2.4. (Case 2B) (supernode)



n－1＝4－1＝3

independent nodal voltages

1 voltage source

Only 2 unknowns are needed.

Say , choose e_a & e_b

4.2 The Node-Voltage Method

Node a : $(e_a - e_b) + (e_a - e_c)4 = 1A - 5A \quad \cdots\cdots \quad (1)$

Let the current through the voltage source be i_x .

Node b : $(e_b - e_a) + e_b(2) + i_x = 0$

Node c : $-i_x + e_c(3) + (e_c - e_a)4 = 5A$

$(e_b - e_a) + e_b(2) + e_c(3) + (e_c - e_a)4 = 5A \quad \cdots\cdots \quad (2)$

4.2 The Node-Voltage Method

Which is the KCL equation written for the supernode including nodes b and c .

From the known voltage source , one can get

$$e_c = e_b - 5V \qquad \mathbf{L\,L\,L\,L\,L\,L\,L\,L\,L\,L\,L} \quad (3)$$

4.2 The Node-Voltage Method

Substituting (3) into (1) & (2)

$$(e_a - e_b) + (e_a - e_b + 5)4 = -4 \qquad \mathbf{L\,L\,L\,L}\,(4)$$

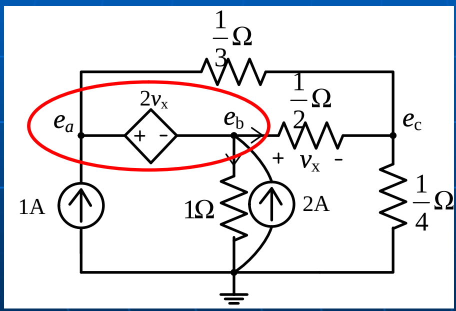
$$(e_b - e_a) + 2e_b + 3(e_b - 5) + (e_b - 5 - e_a)4 = 5 \quad (5)$$

From eqs.(4) and (5)

$$\begin{pmatrix} 5 & -5 \\ -5 & 10 \end{pmatrix}\begin{pmatrix} e_a \\ e_b \end{pmatrix} = \begin{pmatrix} -20-4 \\ 15+20+5 \end{pmatrix}$$

4.2 The Node-Voltage Method

Example 4.2.5 (Case 2C)



n-1=4-1=3, nodal voltages

There is one (dependent) voltage source.

Only two unknown are needed.

Say ,choose e_b and e_c.

4.2 The Node-Voltage Method

For the supernode containing nodes a and b:

$$e_b(1) + (e_b\text{-}e_c)2 + (e_a\text{-}e_c)3 = 1A + 2A \quad \ldots\ldots(1)$$

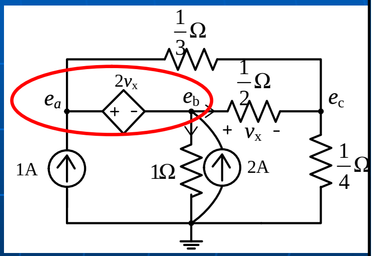
For node c:

$$(e_c\text{-}e_b)2 + e_c(4) + (e_c\text{-}e_a)3 = 0 \quad \ldots\ldots\ldots\ldots\ldots(2)$$

4.2 The Node-Voltage Method

Express the controlling parameters in terms of nodal voltages.

$$2v_x = 2(e_b - e_c)$$



Also, for the known voltage source

$$e_a = 2v_x + e_b = 2(e_b - e_c) + e_b$$

$$= 3e_b - 2e_c \text{ ....(3)}$$

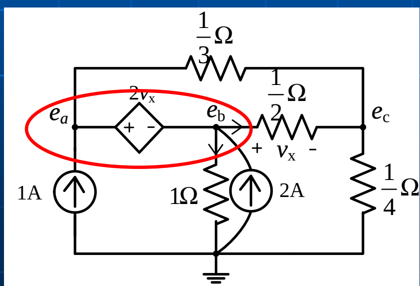
4.2 The Node-Voltage Method

Substituting (3) into (1) and (2) yields

$$e_b + 2(e_b - e_c) + (3e_b - 2e_c - e_c)3 = 3A \text{ ....(4)}$$

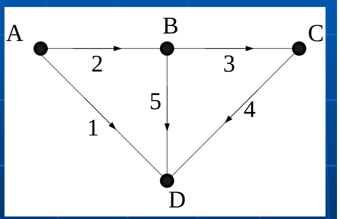
$$(e_c - e_b)2 + 4e_c + (e_c - 3e_b + 2e_c)3 = 0 \text{ ....(5)}$$

$$\begin{bmatrix} \mathbf{12} & \mathbf{-11} \\ \mathbf{-11} & \mathbf{15} \end{bmatrix} \begin{bmatrix} e_b \\ e_c \end{bmatrix} = \begin{bmatrix} \mathbf{3A} \\ \mathbf{0} \end{bmatrix}$$



4.2 The Node-Voltage Method

node-to-branch incidence matrix A_a



	b1	b2	b3	b4	b5
A	1	1	0	0	0
B	0	-1	1	0	1
C	0	0	-1	1	0
D	-1	0	0	-1	-1

4.2 The Node-Voltage Method

$$A_a \triangleq \{a_{ik}\},\ a_{ik} = \begin{cases} 1 & \text{if} \quad \textit{branch i-k leaves node i} \\ -1 & \text{if} \quad \textit{branch i-k enters node k} \\ 0 & \text{if} \quad \textit{branch i-k is not} \\ & \qquad \textit{incident with node i and k} \end{cases}$$

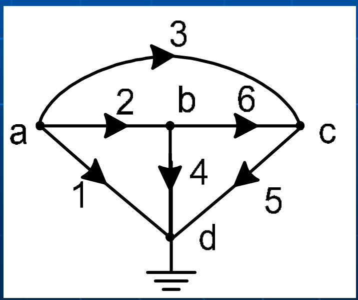
An oriented graph $\overset{Ù}{G}$ can be represented by its incident matrix A_a in computer

4.2 The Node-Voltage Method

Consider the general case

Example 4.2.5 b=5 , n=4

node to branch incidence matrix



Aa =	b_1	b_2	b_3	b_4	b_5	b_6
a	1	1	1	0	0	0
b	0	-1	0	1	0	1
c	0	0	-1	0	1	-1
d	-1	0	0	-1	-1	0

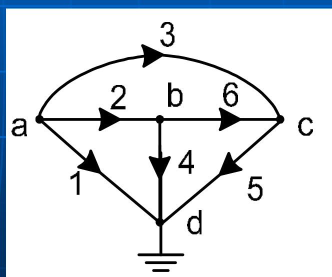
4.2 The Node-Voltage Method

Reduced node to branch incidence matrix

If node d is chosen as datum node

N =

1	1	1	0	0	0
0	-1	0	1	0	1
0	0	-1	0	1	-1



4.2 The Node-Voltage Method

Since each branch voltage can be expressed in terms of nodal voltages：

$V_1 = e_a - 0$ ， Let

$V_2 = e_a - e_b$

$V_3 = e_a - e_c$

$V_4 = e_b - 0$

$V_5 = e_c - 0$

$V_3 = e_b - e_c$

$$V_b^T = [V_1\ V_2\ V_3\ V_4\ V_5\ V_6]$$

$$e^T = [e_a\ e_b\ e_c]$$

Then

$$\boxed{V_b = N^T e} \text{.........(1)}$$

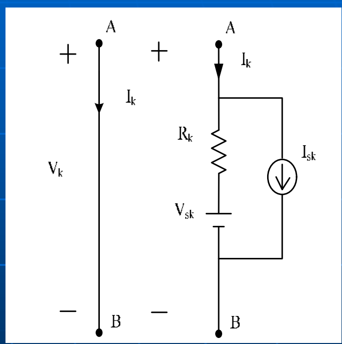
4.2 The Node-Voltage Method

- KCL, N I_b=0..........(2)

$$I_b^T = [I_1\quad I_2\quad I_3\quad I_4\quad I_5\quad I_6]$$

， KVL, automatically satisfied
Example, loop abd

$$v_2 + v_4 - v_1 = 0$$
$$\Rightarrow (\underline{e_a} - e_b) + (e_b - 0) - (\underline{e_a} - 0) = 0$$



4.2 The Node-Voltage Method

- General computer model

$$I_k = I_{sk} + (V_k - V_{sk})G_k$$

$$G_k = \frac{1}{R_k}$$

$$K = 1, 2, \ldots\ldots\ldots, b$$

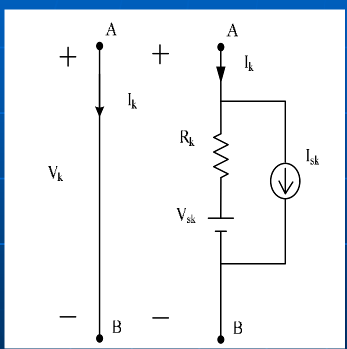
using matrix notation

$$\underline{I}_b = \underline{I}_s + [G](\underline{V}_b - \underline{V}_s) \ldots\ldots\ldots(3)$$

where $\underline{I}_s^T = [I_{s1}\ I_{s2} \cdots I_{sb}]$

$$\underline{V}_s^T = [V_{s1}\ V_{s2} \cdots V_{sb}]$$

4.2 The Node-Voltage Method



Multiply both sides of (3) by N

$$\boldsymbol{0} = \boldsymbol{N}\,\underline{\boldsymbol{I}}_s + \boldsymbol{N}[\boldsymbol{G}](\boldsymbol{V}_b - \boldsymbol{V}_s) \ldots\ldots\ldots\ldots\ldots(4)$$

substitute (1) into

$$\boldsymbol{0} = \boldsymbol{N}\,\underline{\boldsymbol{I}}_s + \underline{\boldsymbol{N}[\boldsymbol{G}]}\boldsymbol{N}^T\underline{\boldsymbol{e}} - \boldsymbol{N}[\boldsymbol{G}]\boldsymbol{V}_s$$

$$\therefore \boldsymbol{N}[\boldsymbol{G}]\boldsymbol{N}^T\underline{\boldsymbol{e}} = \boldsymbol{N}[\boldsymbol{G}]\underline{\boldsymbol{V}}_s - \boldsymbol{N}\,\underline{\boldsymbol{I}}_s\ ,$$

$$\boldsymbol{A}\underline{\boldsymbol{X}} = \underline{\boldsymbol{b}}$$

$$\boldsymbol{A} = \boldsymbol{N}[\boldsymbol{G}]\boldsymbol{N}^T$$

$$\boldsymbol{b} = \boldsymbol{N}[\boldsymbol{G}]\boldsymbol{V}_s - \boldsymbol{N}\boldsymbol{I}_s$$

4.3 The Mesh Current Method

(a) Another general method for analyzing planar circuits.

(b) A planar circuit is one that can be drawn in a plane with no branches crossing one another；otherwise , it is nonplanar.

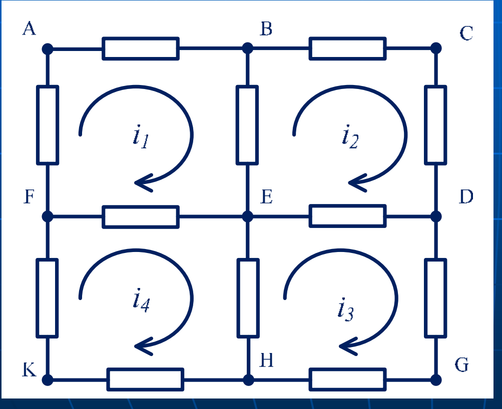
4.3 The Mesh Current Method

(c) A mesh in a planar circuit is a loop that does not contain any other loop within it.

(d) Instead of choosing branch currents as unknown variables , one can choose mesh currents as unknown variables to reduce the number of equations which must be solved simultaneously.

4.3 The Mesh Current Method

Example 4.3.1 :



mesh currents

i_1 : *mesh A B E F A*

i_2 : *mesh B C D E B*

i_3 : *mesh E D G H E*

i_4 : *mesh F E H K F*

4.3 The Mesh Current Method

(e) If the mesh currents are known , then any branch current can be obtained from mesh currents and the corresponding branch voltage can be obtained from the component model directly.

(f) For a planar circuit consists of b branches and n nodes, one can have b-(n-1) *l* independent meshes , where *l* is the number of links.

(g) KCL is automatically satisfied.

4.3 The Mesh Current Method

Mesh Analysis Method

Step 1：Assign mesh currents, i_1, i_2, …i_l as unknowns.

Step 2：Apply KVL to each mesh and use Ohm's law to express the voltages in terms of the mesh currents.

Step 3：Solve the algebraic equation

$$A\underline{x}=\underline{b}$$

and find the desired answer.

4.3 The Mesh Current Method

Examples:

Case 1：Planar circuits with voltage sources

(A) Only independent voltage sources.

： by inspection.

(B) With dependent voltages sources.

： Need one more step, namely express the controlling parameter in terms of the mesh currents.

4.3 The Mesh Current Method

Case 2 ： Planar circuits with current sources

(A) Independent current source exist only in one mesh.

： Trivial case.

(B) Independent current source exists between two meshes.

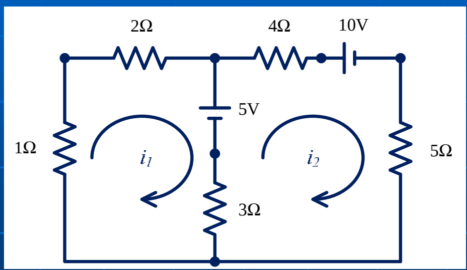
： Use supermesh concept

(C) With dependent current source

： Need one more step, namely express the controlling parameter in terms of the mesh currents.

4.3 The Mesh Current Method

Example 4.3.2 ： Case 1A(Basic case)



$b=7$, $n=6$

$l=7-(6-1)=2$

Choose i_1 & i_2 mesh currents.

mesh 1 ： KVL , $i_1\times 1+i_1\times 2+5V+(i_1-i_2)\times 3=0$

mesh 2 ： KVL , $(i_2-i_1)\times 3+(-5)+i_2\times 4+10V+i_2\times 5=0$

4.3 The Mesh Current Method

Rearrange the above equations

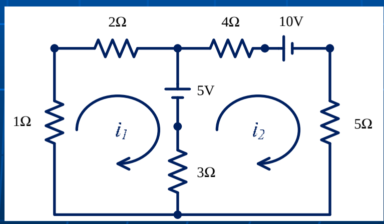
$$\begin{bmatrix} 1+2+3\ \Omega & -3\,\Omega \\ -3\,\Omega & 3+4+5\ \Omega \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} -5V \\ 5-10 \end{bmatrix}$$

$$[R_{ik}]\underline{i} \cong R\underline{i} = \underline{V}_S$$

$\underline{i}$: mesh current vector

$\underline{V}_S$: sourse voltage vector

R : resistance matrix



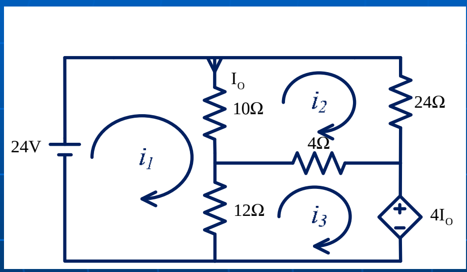
4.3 The Mesh Current Method

R_{kk} = sum of all the resistances in mesh k

R_{ik} = sum of the resistances between meshes i and k and the algebraic sign depends on the relative direction of meshes i and k , plus (minus)sign for same(opposite) direction.

4.3 The Mesh Current Method

Example 4.3.3：Case 1B



Step 1. Choose mesh currents i_1, i_2, i_3

Step 2. Same as example 1

$$\begin{bmatrix} 10+12 & -10 & -12 \\ -10 & 10+4+24 & -4 \\ -12 & -4 & 4+12 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 24 \\ 0 \\ -4I_0 \end{bmatrix}$$

4.3 The Mesh Current Method

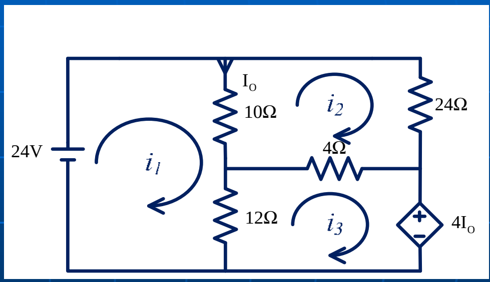
Step 3. Express I_o in terms of mesh currents $I_o = i_1 - i_2$

Substitute into the above mesh equations

$$\begin{bmatrix} 10+12 & -10 & -12 \\ -10 & 10+4+24 & -4 \\ -12+4 & -4-4 & 4+12 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 24 \\ 0 \\ 0 \end{bmatrix}$$

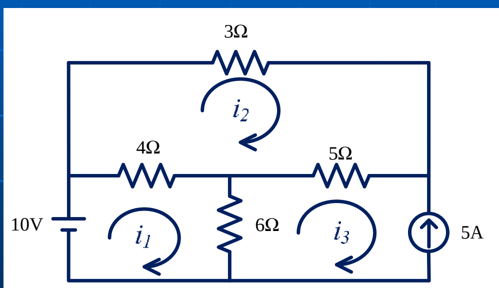
4.3 The Mesh Current Method

$$\therefore\ i_1 = 2.25\text{A}$$
$$i_2 = 0.75\text{A}$$
$$i_3 = 1.5\text{A}$$
$$\text{I}_\text{O} = i_1 - i_2 = 1.5\text{A}$$



4.3 The Mesh Current Method

Example 4.3.4：Case 2A



b= 6, n=4

l= 6-(4-1)=3

Step 1. Choose mesh currents i_1, i_2, i_3

Step 2. For case 2A i_3=-5A, answer

4.3 The Mesh Current Method

KVL for mesh 1：

$$(i_1 - i_2)\times 4 + (i_1 - i_3)\times 6 = 10\text{V}$$

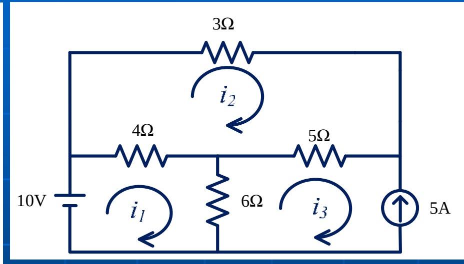
KVL for mesh 2：

$$i_2 \times 3 + (i_2 - i_3)\times 5 + (i_2 - i_1)\times 4 = 0$$

Substitute $i_3 = -5\text{A}$ into KVL equations

$$(i_1 - i_2)\times 4 + (i_1 + 5)\times 6 = 10\text{V}$$

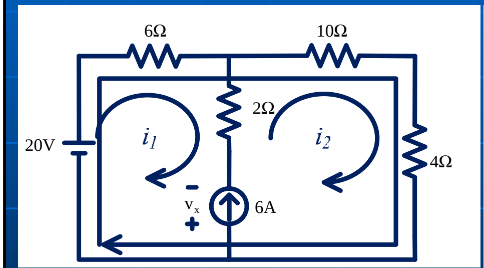
$$i_2 \times 3 + (i_2 + 5)\times 5 + (i_2 - i_1)\times 4 = 0$$



$$\begin{bmatrix} 4+6 & -4 \\ -4 & 3+5+4 \end{bmatrix} \Omega \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 10-30 \\ -25 \end{bmatrix}$$

4.3 The Mesh Current Method

Example 4.3.5：Case 2B (Supermesh concept)



Step 1. Choose mesh currents

i_1 , i_2

Step 2. Algebraic solution approach：

Assume the voltage across the current source is v_x

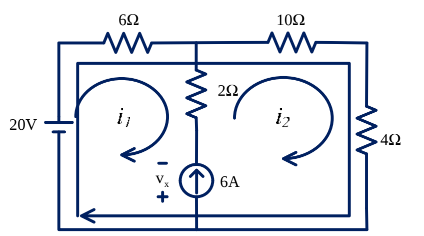
4.3 The Mesh Current Method

KVL for mesh 1：

$$i_1 \times 6 + \underline{\underline{(i_1 - i_2) \times 2 + (-v_x)}} = 20\text{V}$$

KVL for mesh 2：

$$\underline{\underline{v_x + (i_2 - i_1) \times 2}} + i_2 \times 10 + i_2 \times 4 = 0$$



Add two KVL equations

$$i_1 \times 6 + i_2 \times 10 + i_2 \times 4 = 20\text{V}.......(A)$$

This is the KVL equation for the supermesh!

Need one more equation form the current source：

$$6\text{A} = i_2 - i_1...............................(B)$$

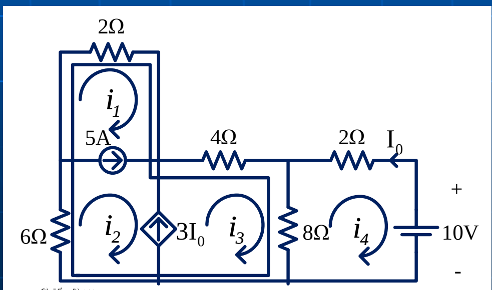
From (A) and (B) , one can get the solution.

4.3 The Mesh Current Method

Example 4.3.6：Case 2C

Step1: choose i_1, i_2, i_3, i_4 mesh currents.

There are two current sources , one is independent and the other is dependent. And the number of unknowns can be reduced by 2 , namely only two equations are required.

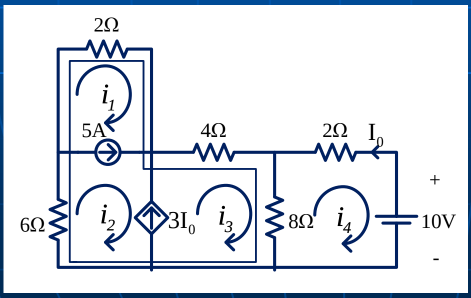


4.3 The Mesh Current Method

Step2: We can write KVL equations for mesh 4 and the supermesh as indicated in Fig.

mesh 4: $(i_4 - i_3)*8 + i_4*2 = -10$ V (A)

supermesh: $i_2*6 + i_1*2 + i_3*4 + (i_3-i_4)*8 = 0$ (B)



4.3 The Mesh Current Method

For two current sources, we can get two equations to eliminate two unknowns

$5A = i_2 - i_1 \quad => \quad i_2 = i_1 + 5$ (C)

$3I_0 = i_2 - i_3 = i_1 + 5 - i_3 \quad => \quad i_3 = i_1 + 5 - 3I_0$ (D)

4.3 The Mesh Current Method

For this case with dependent source, we need one more step, namely to express the controlling parameter I_o in terms of mesh currents.

$$I_0 = -i_4 \qquad \text{(E)}$$

Substitute (C), (D), and (E) into (A) and (B) :

$$[i_4 - (i_1 + 5 + 3i_4)]*8 + i_4*2 = -10$$

$$(i_1 + 5)*6 + i_1*2 + (i_1 + 5 + 3i_4)*4 + [(i_1 + 5 + 3i_4) - i_4]*8=0$$

4.3 The Mesh Current Method

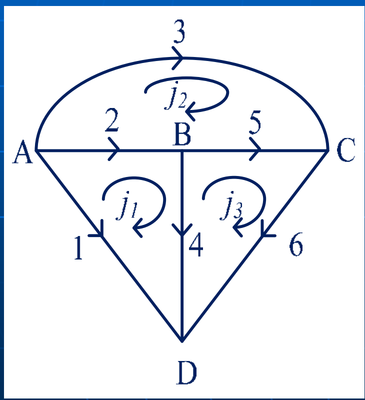
Equivalently

$$\begin{bmatrix} -8 & -14 \\ 28 & 36 \end{bmatrix} \begin{bmatrix} i_1 \\ i_4 \end{bmatrix} = \begin{bmatrix} 30 \\ -90 \end{bmatrix}$$

$$\therefore\ i_1 = -7.5 \ \text{A}$$
$$i_2 = -2.5 \ \text{A}$$
$$i_3 = 3.93 \ \text{A}$$
$$i_4 = 2.143 \ \text{A}$$

4.3 The Mesh Current Method

(i) General Case



	b1	b2	b3	b4	b5	b6
mesh1	-1	1	0	1	0	0
mesh2	0	-1	1	0	-1	0
mesh3	0	0	0	-1	1	1
outer mesh	1	0	-1	0	0	-1

4.3 The Mesh Current Method

M_{ik} = 1 (-1) if branch k is included in mesh i and the branch current direction is in the same (opposite) direction as the mesh current.

M_{ik} = 0 if branch k is not included in mesh i.

Reduced mesh to branch matrix M

M =

-1	1	0	1	0	0
0	-1	1	0	-1	0
0	0	0	-1	1	1

4.3 The Mesh Current Method

- KCL is automatically satisfied

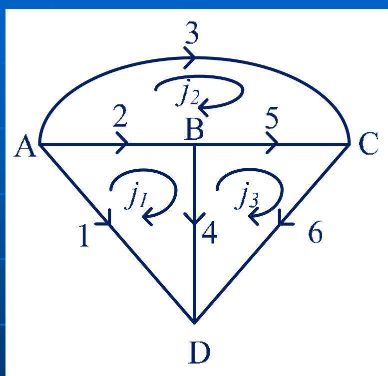
, KVL

$M\underline{V}_B = 0$ (A)

$\underline{V}_B = [\ v_1\ \ v_2\ \ v_3\ \ v_4\ \ v_5\ \ v_6\ ]^T$

$$\begin{bmatrix} -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

4.3 The Mesh Current Method



$-v_1 + v_2 + v_4 = 0$: KVL for mesh 1

$-v_2 + v_3 - v_5 = 0$: KVL for mesh 2

$-v_4 + v_5 + v_6 = 0$: KVL for mesh 3

4.3 The Mesh Current Method

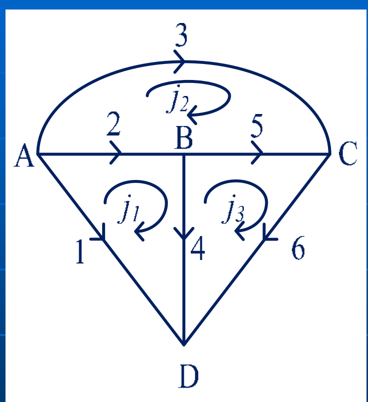
The branch currents can be expressed in terms of mesh currents

$$\underline{I}_B = M^T \underline{J} \qquad (B)$$

$$\underline{I}_B \quad [\, i_1 \;\; i_2 \;\; i_3 \;\; i_4 \;\; i_5 \;\; i_6 \,]^T$$

$$\underline{J} = [\, j_1 \;\; j_2 \;\; j_3 \,]^T$$

4.3 The Mesh Current Method



$i_1 = -j_1$

$i_2 = j_1 - j_2$

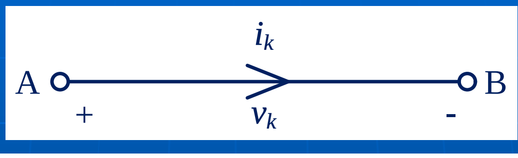
$i_3 = j_2$

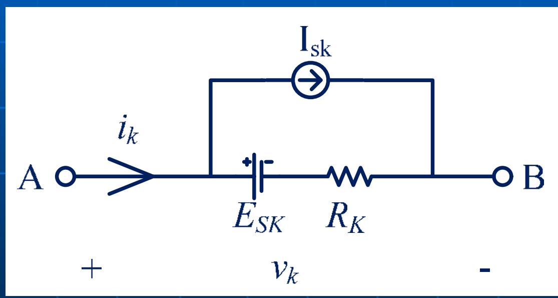
$i_4 = j_1 - j_3$

$i_5 = -j_2 + j_3$

$i_6 = j_3$

4.3 The Mesh Current Method





$$v_k = E_{SK} + (i_k - I_{sk})\, R_K$$

4.3 The Mesh Current Method

In matrix form

$\underline{V}_B = [R]\underline{I}_B + \underline{E}_S - [R]\underline{I}_S$ (C)

$[R] = \text{diag}\,[R_1\ R_2\ \ldots\ldots\ R_b]$ branch resistance matrix

$\underline{E}_S = [E_{S1}\ E_{S2}\ \ldots\ldots\ E_{Sb}]^T$ branch voltage source vector

$\underline{I}_S = [I_{S1}\ I_{S2}\ \ldots\ldots\ I_{Sb}]^T$ branch current source vector

4.3 The Mesh Current Method

Multiply M on both sides of (C) and use relation (A) to get

$$\underline{0} = M\underline{V}_B = M[R]\,\underline{I}_B + M\,\underline{E}_S - M[R]\underline{I}_S \qquad \text{(D)}$$

Substitute (B) into (D)

$$M[R]M^T\underline{J} = M[R]\underline{I}_S - M\underline{E}_S$$

$$A\underline{X} = \underline{b}$$

4.3 The Mesh Current Method

(j) Nodal Versus Mesh Analysis

Both methods provide a systematic way of analyzing a complex network.

Which method should be chosen for a particular problem? The choice is based on two considerations.

(1)First consideration: choose the method which contains minimum number of unknowns that requires simultaneous solution.

4.3 The Mesh Current Method

(1) First consideration: choose the method which contains minimum number of unknowns that requires simultaneous solution.

For a circuit contains b branches, n nodes, among them there are m_v voltage sources and m_i current sources, including dependent source, then the number of unknowns to be solved simultaneously is

$n\text{-}l\text{-}m_v$: for nodal analysis

$b\text{-}(n\text{-}1)\text{-}m_i$: for mesh analysis

4.3 The Mesh Current Method

(2)Second consideration: Depends on the solution required, if nodal voltages are required, it may be expedient to use nodal analysis. If branch or mesh currents are required it may be better to use mesh analysis.

4.4 Fundamental Loop Analysis

- Two other variations besides nodal and mesh analysis are introduced, namely the fundamental loop analysis and the fundamental cutset analyses.
- They are useful for understanding how to write the state equation of a circuit.

4.4 Fundamental Loop Analysis

Theorem

Given an oriented graph G of n nodes and b branches, and a tree T of G, then

1. Between any pair of nodes there is a unique path along the tree.
2. There are n-1 tree branches and b-n+1 links.

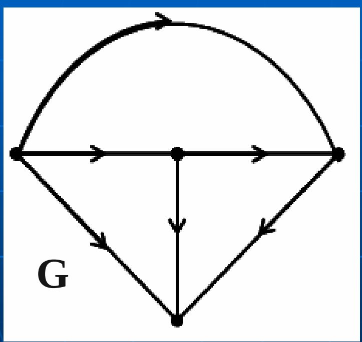
4.4 Fundamental Loop Analysis

Theorem

3. Every link of T and the corresponding unique tree path between its two terminals constitute a unique loop, called the fundamental loop associated with this link.

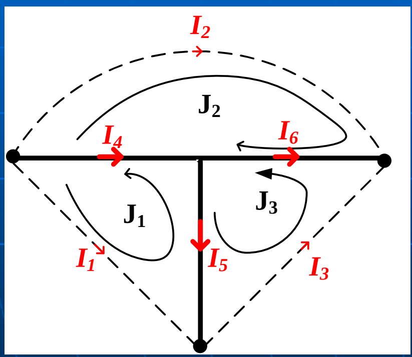
4. Every tree branch of T together with some links defines a unique cutset of G, called the fundamental cutset associated with this tree branch.

4.4 Fundamental Loop Analysis



Given the oriented graph G of n (= 4) nodes and b (= 6) branches.

4.4 Fundamental Loop Analysis



Step 1

Pick an arbitrary tree T. Number the links from 1 to l and then tree branches from l +1 to b.

4.4 Fundamental Loop Analysis

Step 2

Choose link currents J1, J2, J3 as unknowns and adopt each link current direction as the reference of the corresponding fundamental loop current.

Write down KVL equations for the **l** fundamental loops.

4.4 Fundamental Loop Analysis

$$
\begin{array}{c}
Loop1 \\ Loop2 \\ Loop3
\end{array}
\begin{bmatrix}
1 & 0 & 0 & -1 & -1 & 0 \\
0 & 1 & 0 & -1 & 0 & -1 \\
0 & 0 & 1 & 0 & 1 & -1
\end{bmatrix}
\begin{bmatrix}
V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6
\end{bmatrix}
=
\begin{bmatrix}
0 \\ 0 \\ 0
\end{bmatrix}
$$

branch voltage vector $V_B^T @ [V_1 \quad V_2 \quad V_3 \quad V_4 \quad V_5 \quad V_6]$

branch current vector $I_B^T @ [I_1 \quad I_2 \quad I_3 \quad I_4 \quad I_5 \quad I_6]$

In matrix form $L V_B = 0 \; .... \; (A)$

4.4 Fundamental Loop Analysis

Step 3 ***KCL***

If every branch current is formed by linear combination of loop currents, then KCL will be satisfied automatically.

$$
\begin{aligned}
I_1 &= J_1 \\
I_2 &= J_2 \\
I_3 &= J_3 \\
I_4 &= -J_1 - J_2 \\
I_5 &= -J_1 + J_3 \\
I_6 &= -J_2 - J_3
\end{aligned}
$$

In matrix form

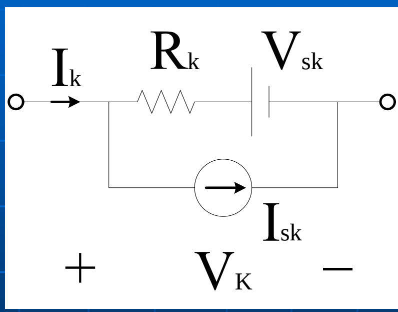
$$I_B = L^T J \; .... (B)$$

$J^T = [J_1 \quad J_2 \quad J_3]$ Link current vector

4.4 Fundamental Loop Analysis

Step 4 *Component model*

branch k



$$\therefore \quad V_k = (I_k - I_{sk})R_k + V_{sk} \;, k = 1,2,\ldots,b$$

$$or \;\; V_B = [R](I_B - I_s) + V_s \quad \ldots\ldots\ldots\ldots\ldots\ldots \; (C)$$

4.4 Fundamental Loop Analysis

Multiply (C) by L matrix and substitute (A) and (B)

$$L V_B = L[R] I_B \; - L[R] I_s + L V_s$$

$$0 = L[R] L^T J - L[R] I_s + L V_s$$

$$\therefore \quad L[R]L^T J = L[R] I_s - L V_s$$

$$\Rightarrow Z\,J \triangleq e_s \quad \ldots\ldots\ldots\ldots\ldots \; (D)$$

[R] ： diagonal matrix

Z ： $l \times l$ matrix, called loop impedance matrix

e_s ： equivalent loop voltage source vector

4.4 Fundamental Loop Analysis

Step 5

Solve equation (D) and calculate I_B and V_B.

Note that

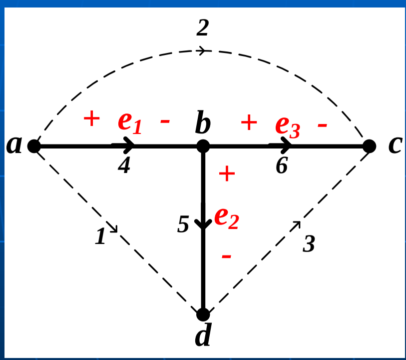
1. Link currents are used as the unknowns.
2. The number of links is the same as that of the mesh currents. Because they are all optimal methods in terms of using minimum number of unknown current variables.

4.4 Fundamental Loop Analysis

3. The resulting circuit loop equations depend on the choice of a tree.
4. Compared with mesh analysis, the fundament loop analysis is more general, i.e. not restricted to planar circuits, but requires defining a tree.

4.5 Fundamental Cutset Analysis

Use the same example as an demonstration.



Given the oriented graph G.

Step 1

Pick a tree T. Number the links from 1 to l and then tree branches from l +1 to b.

4.5 Fundamental Cutset Analysis

Step 2

Choose the tree branch voltages e_1, e_2, e_3 as unknowns and adopt the tree branch current as the reference direction of the associated fundamental cutset. Write down KCL equations for the n-1 fundamental cutsets.

Cutset 1 for branch 4 $\Rightarrow$ $\boxed{i_4 + i_1 + i_2 = 0}$

Cutset 2 for branch 5 $\Rightarrow$ $\boxed{i_5 + i_1 - i_3 = 0}$

Cutset 3 for branch 6 $\Rightarrow$ $\boxed{i_6 + i_2 + i_3 = 0}$

4.5 Fundamental Cutset Analysis

In matrix form

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C\, I_B = 0 \;\; \dots\dots\dots\dots\dots\dots \;(A)$$

4.5 Fundamental Cutset Analysis

Step 3 ***KVL***

Every branch voltage can be expressed in terms of the tree branch voltages.

In matrix form

$$V_B = C^T e \;\; \dots\dots\dots\dots\dots\dots\dots (B)$$

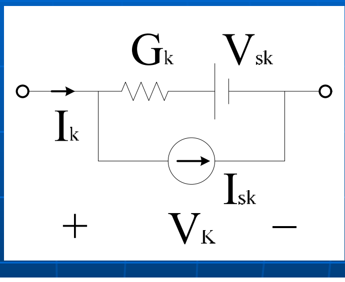
$$e^T = \begin{bmatrix} e_1 & e_2 & e_3 \end{bmatrix}$$

Tree branch voltage vector

4.5 Fundamental Cutset Analysis

Step 4 ***Component model***

branch k



$$I_k = G_k\,(V_k - V_{sk}) + I_{sk}\ ,k = 1,2,3,\ldots ,b$$

In matrix form

$$I_B = [G](V_B - V_s) + I_s \;\; \ldots\ldots\ldots\ldots\ldots\ldots \;\; (C)$$

4.5 Fundamental Cutset Analysis

From (A), (B) and (C)

$$C\,I_B = C[G]V_B \;\; - C[G]V_s + C\,I_s$$

$$0 \;\; = C[G]C^T e - C[G]V_s + C\,I_s$$

$$C[G]C^T e = C[G]V_s - C\,I_s$$

$$\Rightarrow \; Y\,e \triangleq J_s \;\; \ldots\ldots\ldots\ldots\ldots\ldots\ldots \;\; (D)$$

Y ： cutset admittance matrix
[G] ： diagonal conductance matrix
J_s ： cutset current source vector

4.5 Fundamental Cutset Analysis

Step 5

Solve equation (D) and calculate I_B and V_B.

Note that

1. Tree branch voltages are used as unknowns.
2. The number of tree branches is the same as that of the nodal analysis. Because they are all optimal methods in terms of using minimum number of unknown voltages.

4.5 Fundamental Cutset Analysis

3. The resulting cutset equations depend on the choice of a tree..
4. Compared with nodal analysis, the fundament loop method requires defining a tree.

Summary

Table 1 Comparison of Nodal and Fundamental Cutset Analyses

Methods	Nodal Analysis	Fundamental Cutset Analysis
KCL	$NI_B = 0$	$CI_B = 0$
KVL	$V_B = N^T e$	$V_B = C^T e_t$
Component Model	$I_B = I_S + [G](V_B - V_S)$	$I_B = I_S + [G](V_B - V_S)$
Equation to be Solved	$Ax = b$	$Ax = b$
	$A = N[G]N^T$ $b = N[G]V_S - NI_S$ $x = e$	$A = C[G]C^T$ $b = C[G]V_S - CI_S$ $x = e_t$

e : node voltage vector

e_t : tree branch voltage vector

Summary

Table 2 Comparison of Mesh and Fundamental Loop Analyses

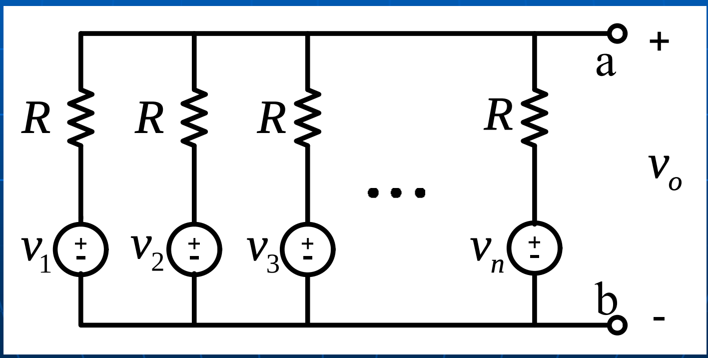
Methods	Nodal Analysis	Fundamental Cutset Analysis
KCL	$I_B = M^T J_m$	$I_B = L^T J$
KVL	$MV_B = 0$	$LV_B = 0$
Component Model	$V_B = E_S + [R](I_B - I_S)$	$V_B = V_S + [R](I_B - I_S)$
Equation to be Solved	$Ax = b$	$Ax = b$
	$A = M[R]M^T$ $b = M[R]I_S - ME_S$ $x = J_m$	$A = L[R]L^T$ $b = L[R]I_S - LV_S$ $x = J$

J_m : mesh current vector

J : link current vector

Summary

An interesting and practical circuit
[Problem 4.16]



Summary

By using node-voltage method:
Choose node b as the datum node .

Then

$$\frac{v_o - v_1}{R} + \frac{v_o - v_2}{R} + \frac{v_o - v_3}{R} + \ldots + \frac{v_o - v_n}{R} = 0$$

$$\therefore n\, v_o = (v_1 + v_2 + v_3 + \ldots + v_n)$$

$$\therefore v_o = \frac{1}{n} \sum_{k=1}^{n} v_k$$

Summary

It is an averaging circuit used to find the average value of n voltages.

Note that there is a common node and all resisters are equal , called sharing bus method.

Summary

- Objective 1 : Understand and be able to use the node-voltage method to solve a circuit.
- Objective 2 : Understand and be able to use the mesh-current method to solve a circuit.
- Objective 3 : Be able to decide whether the node-voltage method or the mesh –current method is preferred approach to solving a particular circuit.

Summary

- Objective 4 : Understand how a circuit is stored in a computer and some basic graph theory.
- Objective 5 : Understand the basic principle of fundamental loop analysis.
- Objective 6 : Understand the basic principle of fundamental cutset analysis.

Summary

- Problem : 4.9
  4.17
  4.24
  4.31
  4.39
  4.43
- Due within one week.