1. Given a signal \( s(t) = e^{-|t|/\tau} \), let \( s_1(t) = \begin{cases} s(t) & |t| \leq T \\ 0 & \text{elsewhere} \end{cases} \)
and \( s_2(t) = s_1(t) \cdot \left[ \sum_{n=-\infty}^{\infty} \delta(t - nT) \right] \).

(a) (3%) Derive and plot \( S(f) \), the Fourier transform of \( s(t) \).
(b) (3%) Derive and plot \( S_1(f) \), the Fourier transform of \( s_1(t) \).
(c) (3%) Derive and plot \( S_2(f) \), the Fourier transform of \( s_2(t) \).
(d) (3%) How should you choose \( T \) and \( T_1 \) in order to make \( S_2(f) \) in baseband as close to \( S(f) \) as possible? Why?

2. The characteristic function of a random variable \( X \) is defined as \( M_x(\lambda) = E[e^{i\lambda X}] \).

(a) (5%) Find the characteristic function of a Gaussian random variable \( Y \) with mean \( \mu \) and variance \( \sigma^2 \).
(b) (5%) Apply the result obtained in (a) to find the 2nth center moment of \( Y \) defined by \( E[(Y - \mu)^{2n}] \).

3. AWGN is assumed in the following.

(a) (5%) Draw the block diagram of a coherent double-sideband (DSB) demodulator and find out the demodulated output SNR.
(b) (5%) Write out the FM modulated signal for stereophonic FM broadcasting in terms of the carrier frequency \( f_c \), \( m_L(t) + m_R(t), m_L(t) - m_R(t) \), subcarrier frequency \( f_{sc} \), and pilot frequency \( f_p \), where \( m_L(t) \) and \( m_R(t) \) are the left and right channel audio messages respectively.
(c) (5%) Show that the FM demodulated output noise power spectral density is proportional to \( f^2 \).
(d) (5%) Find the ratio between the demodulated output SNR for message \( m_L(t) - m_R(t) \) and the demodulated output SNR for message \( m_L(t) + m_R(t) \).
(e) (5%) Draw the block diagram of a superheterodyne stereophonic FM broadcasting receiver. Explain how to avoid the image signal.

4. One of two equally likely signals is transmitted and received in additive Gaussian noise \( n(t) \). The signals are \( s_1(t) = 4 = -s_2(t) \), the bit period being 1 msec long. Eight equally spaced independent samples of the received signal \( r(t) = \{ s_1(t) \text{ or } s_2(t) \} + n(t) \) are taken and constitute an 8-dimensional vector \( \mathbf{y} = [y_1, y_2, \ldots, y_8] = [s_1 \text{ or } s_2] + n \), where \( s_1 = [4, 4, 1, \ldots, 4] = -s_2 \), \( n = [n_1, n_2, \ldots, n_8] \), and \( n_i \) 's, \( i = 1, 2, \ldots, 8 \), are independent zero-mean Gaussian noise with variance \( \sigma^2 \).

(a) (4%) For the observed data \( \mathbf{y} \), what is the optimum detector?
(b) (4%) According to (a), if \( \mathbf{y} = [4.5, 0, -1.5, 2, -6, 10, 1, -4] \), which signal would you decide?
(c) (7%) What is the probability of error for the optimum detector in (a)?
5. (10%) A sample of a polar signal of amplitude ±1 is perturbed by a random noise \( n \) with probability density function \( f(n) = \frac{3}{32}(4 - n^2), \ |n| \leq 2 \). Find the minimum probability of error if the a priori probabilities are \( P_{+1} = \frac{3}{5} = 1 - P_{-1} \). What is the decision threshold?

6. Consider coherent BPSK and QPSK transmission systems in AWGN \( (S_n(f) = N_0/2) \) and the received signals are optimally detected.

(a) (4%) If both systems transmit at the same data rate \( R \) and the same transmission power \( P \), write out their bit error probabilities. Which system is better? Why?

(b) (4%) If both systems transmit at the same null-to-null bandwidth \( B \) and the same transmission power \( P \), write out their bit error probabilities. Which system is better? Why?

(c) (5%) Derive and plot the power spectral density of a BPSK modulated signal at a data rate of \( R \) bits/sec.

7. **Convolutional Code Encoder**

(a) (5%) Define the encoder state and depict the corresponding encoder state diagram.

(b) (5%) What is the resulting codeword if the input sequence = \( (1 \ 1 \ 0 \ 0 \ 0 \ 1 \ 0 \ 1) \).

8. (5%) Find a ternary (i.e., code symbols are 0, 1, and 2) Huffman code for the following source:

<table>
<thead>
<tr>
<th>Source Alphabet</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>.275</td>
</tr>
<tr>
<td>( B )</td>
<td>.225</td>
</tr>
<tr>
<td>( C )</td>
<td>.2</td>
</tr>
<tr>
<td>( D )</td>
<td>.175</td>
</tr>
<tr>
<td>( E )</td>
<td>.125</td>
</tr>
</tbody>
</table>