1. (12 pt) Consider a sinusoidal signal with random phase, defined as
\[ x(t) = A \cdot \cos (2\pi f_c t + \Theta) \]
where \( A \) and \( f_c \) are constants and \( \Theta \) is a uniformly distributed random variable over the interval \([-\pi, \pi]\), given by
\[ f_{\Theta}(\theta) = \begin{cases} \frac{1}{2\pi}, & -\pi \leq \theta \leq \pi \\ 0, & \text{elsewhere} \end{cases} \]

(a) (6 pt) Find the autocorrelation function of \( x(t) \).
(b) (6 pt) Find the power spectral density of \( x(t) \).

2. (22 pt) Consider a square-law detector whose transfer characteristic is defined as
\[ y(t) = a_1 x(t) + a_2 x^2(t) \]
where \( a_1 \) and \( a_2 \) are constants, \( x(t) \) is the input signal, and \( y(t) \) is the output signal. Assume that the input signal \( x(t) \) is a modulated signal, given by
\[ x(t) = A_c (1 + k_m m(t)) \cos (2\pi f_c t) \]
where \( m(t) \) is a baseband message signal, \( A_c \) and \( k_m \) are constants, and \( f_c \) is the carrier frequency.

(a) (4 pt) What is the modulation scheme used for the input signal \( x(t) \).
(b) (6 pt) Plot the spectrum of the modulated signal \( x(t) \), if the spectrum of the message is given by \( M(f) = \begin{cases} 1, & -W \leq f \leq W \\ 0, & \text{otherwise} \end{cases} \).
(c) (6 pt) Evaluate the output signal \( y(t) \).
(d) (6 pt) Find the condition for which the message signal can be recovered from \( y(t) \).

3. (30 pt) Consider a binary pulse-code modulation signal with signaling interval \( 0 \leq t \leq 1 \), given by
\[ s(t) = \begin{cases} +A \cdot g(t), & \text{if symbol "1" was sent} \\ -A \cdot g(t), & \text{if symbol "0" was sent} \end{cases} \]
where the pulse shaping function \( g(t) \) is given by
\[ g(t) = \begin{cases} 1, & 0 \leq t \leq 1 \\ 0, & \text{otherwise} \end{cases} \]
Assume the channel noise is modeled as additive white Gaussian noise \( w(t) \) with zero mean and power spectral density \( N_0/2 \). Consider a receiver structure used to perform decision making for the received signal \( x(t) = s(t) + w(t) \) in Figure 1.
(a) (4 pt) Explain the advantage of adopting the matched filter.
(b) (6 pt) Find the matched filter with respect to the pulse shaping function.
(c) (6 pt) Sketch the matched filter output with respect to the pulse shaping function.
(d) (8 pt) Calculate the error probability, conditional on sending symbol “0”, with respect to the decision threshold \( \lambda \).
(e) (6 pt) Find the optimal decision threshold \( \lambda_{opt} \) for achieving the minimum symbol error probability, when symbols “0” and “1” are sent with an equal probability.

\[
\text{Matched filter Decision device (St)}
\]

\[
\text{Sample at time } t=1
\]

\[
\text{Say 1, if } y>\lambda
\]

\[
\text{Say 0, if } y<\lambda
\]

\[
\text{Threshold } \lambda
\]

Figure 1

4. (36 pt) Consider optimum data detection for quadrature phase-shift keying (QPSK) with the constellation in Figure 2, where the signal vectors are given by \( s_1 = [+1,+1]^T \), \( s_2 = [-1,+1]^T \), \( s_3 = [-1,-1]^T \), and \( s_4 = [+1,-1]^T \).

(a) (6 pt) Label the QPSK constellation with Gray mapping.
(b) (6 pt) Define the likelihood function and describe the maximum likelihood (ML) decision rule.
(c) (6 pt) Show that the ML decision rule is equivalent to a minimum distance decision rule.
(d) (6 pt) Find and draw the optimum decision regions for QPSK.
(e) (6 pt) Describe the maximum a posteriori probability (MAP) decision rule.
(f) (6 pt) Explain why the MAP decision rule is usually a better choice than the ML decision rule, and under what condition the two decision rules are equivalent.

\[
\text{Figure 2}
\]
Part I. (50%) Please choose the right answer for each question below:

1. (5%) Which of the following has the highest speed to be accessed? (A) main memory (B) cache (C) hard disk
2. (5%) For a disk system, which of the following indicates the sum of the seek time and rotation delay? (A) transfer rate (B) access time (C) latency time
3. (5%) Which of the following is the hexadecimal notation of the bit pattern “01001011”? (A) 4A (B) 4B (C) 4C
4. (5%) For the base ten representation -17, which of the following is the equivalent two’s complement form using patterns of 8 bits? (A) 10010101 (B) 10101101 (C) 11101111
5. (5%) The result of XORing the patterns 10011010 and 11001001 is (A) 01010011 (B) 10001000 (C) 11011011
6. (5%) Which of the following is not a component of an operating system? (A) shell (B) file manager (C) utility (D) dispatcher
7. (5%) Lempel-Ziv-Welsh (LZW) encoding belongs to the group of (A) dictionary encoding (B) lossy schemes (C) differential encoding
8. (5%) The operation JUMP of machine instructions is classified into (A) the data transfer group (B) the arithmetic/logic group (C) the control group
9. (5%) Which of the following contains the address of the next instruction to be executed by the CPU? (A) general-purpose register (B) instruction register (C) program counter
10. (5%) The boot loader is stored in a machine’s (A) read-only memory (B) volatile memory (C) hard disk
Part II. (50%) Please explain the following items clearly. Your score will depend on the correctness and completeness of your answers.

1. (10%) Direct memory access (DMA).
2. (10%) Virtual memory.
3. (10%) Carrier Sense, Multiple Access with Collision Detection (CSMA/CD)
4. (10%) Peer-to-peer (P2P) communication.
5. (10%) Public-key encryption.
1. (20%) Consider the circuit shown as the below, in which the diode cut-in voltages are \( V_r = 0.6V \). Plot \( v_o \) versus \( v_i \) for \( 0 \leq v_i \leq 10V \).

2. (25%) Derive \( v_o \) in relation of \( v_{i1}, v_{i2}, v_{i3}, \) and \( v_{i4} \).

3. (25%) The transistor parameters are \( \beta = 100 \) and \( V_A = \infty \).
   (a) Determine \( I_{cQ} \) and \( V_{eQ} \). (10%)
   (b) Find the small-signal voltage gain \( A_v = v_o / v_s \). (15%)
4. (30%) The transistor has parameters $\beta = 120, V_{BE(ON)} = 0.7V$, and $V_A = 80V$.
   (a) If $R_2 = 85k\Omega$, find the resistance $R_1$ such that $I_{CQ} = 1mA$. (10%)
   (b) Determine the output resistance $R_o$. (10%)
   (c) What is the lower 3 dB corner frequency? (10%)