Part 1: Filling in (60 points, each 5 points)

1. An analog random signal source has an output described by the probability density function

\[ f_x(x) = \begin{cases} \frac{x}{2}, & 0 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases} \]

This source is sampled and quantized into 4 levels using the 3 quantizing boundaries of \( x_k = 0.5k \), \( k = 1, 2, 3 \). The resulting levels are encoded using a Huffman code.
(a) The average information carried in each quantization-output is \( \text{(1)} \).
(b) After Huffman encoder, the average bit-length for each quantization-output is \( \text{(2)} \).
(c) The coding efficiency of the Huffman code is \( \text{(3)} \). (5%)

2. A signal \( x(t) = 2 \cos 2000 \pi t \) is quantized by a uniform quantizer with dynamic range (-4, 4). The output of the quantizer is modulated by polar NRZ code and transmitted through a channel with one-sided mainlobe bandwidth of 20KHz. The quantization noise is assumed to be uniformly distributed.
(a) The maximum number of quantum steps of the quantizer without aliasing distortion is \( \text{(4)} \).
(b) The signal to quantization noise ratio (in dB) of the quantizer's output is \( \text{(5)} \).

3. The power spectral density of a random process \( x(t) \) is shown in Fig. 1.
(a) Express the autocorrelation function \( R_x(\tau) \) as \( \text{(6)} \).
(b) The dc power contained in \( x(t) \) is \( \text{(7)} \).
(c) The ac power contained in \( x(t) \) is \( \text{(8)} \).
(d) To have uncorrelated samples of \( x(t) \), the possible sampling rates are \( \text{(9)} \).

![Fig. 1.](image-url)
4. A superheterodyne receiver operates in the frequency range of 700-2500 KHz.
   The IF frequency \( f_{IF} \) and the local oscillator frequency \( f_{LO} \) are chosen such that \( f_{IF} < f_{LO} \).
   It is required that the image frequencies must fall outside of the 700-2500 KHz region.
   (a) The minimum required \( f_{IF} \) is ______ (10).
   (b) The range of the corresponding \( f_{LO} \) is ______ (11).

5. Through a channel of raised cosine spectrum with roll-off factor \( \alpha = 33.3\% \) and bandwidth = 10 MHz,
   the maximum transmission data rate for OQPSK modulation scheme is ______ (12).

Part 2: (40 分)
1. In an AWGN channel with a noise power spectral density of \( N_0/2 \), two equally likely messages are
   transmitted by
   \[
   s_1(t) = \begin{cases} \frac{A t}{T} & 0 \leq t \leq T \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad s_2(t) = \begin{cases} A - \frac{A t}{T} & 0 \leq t \leq T \\ 0 & \text{otherwise} \end{cases}
   \]
   (a) Determine \( E_b \), the bit energy. (5%)
   (b) Depict the optimal receiver and determine the threshold value for the receiver. (5%)
   (c) With the optimal receiver, determine the bit-error-rate (BER) in terms of Q-function
      \[
      Q(u) = \int_{u}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2} dx
      \]
      and parameters \( A \), \( T \), and \( N_0 \). (5%)
   (d) Known that \( E_b/N_0 = 10.5 \text{dB} \) is required to get \( \text{BER}=10^{-6} \) for coherent QPSK signal, what is the
      required \( E_b/N_0 \) (in dB) for this system to get \( \text{BER}=10^{-6} \)? (5%)

2. The output of a \((3, 1, 2)\) convolutional code are determined by
   \[
   v_i^{(1)} = u_i + u_{i-1} + u_{i-2}, \quad v_i^{(2)} = u_i + u_{i-2}, \quad \text{and} \quad v_i^{(3)} = u_i + u_{i-1}, \quad \text{where} \ \{u_i\} \ \text{is the input message sequence.}
   \]
   (a) Draw the encoder of this code. (5%)
   (b) Draw the state-transition diagram of this code. (5%)
   (c) Draw the trellis diagram for this code. (5%)
   (d) If the input message is \([1 \ 0 \ 0 \ 1 \ 1 \ 0]\), what is the transmitted (encoded) sequence? (5%)